RESOLVING INTERNAL UNDERCUTS OF PARTS IN MOULD DESIGN

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DECLARATION

I hereby declare that this MPhil. thesis titled "Resolving Internal Undercuts of Parts in Mould Design" is composed by myself and describes my own work. I also declare that the work reported in this thesis has not been previously included in a thesis, dissertation or report submitted to this University or any other institutions for a degree, diploma, or other qualification.

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ABSTRACT

The plastics and mould industry has been growing very rapidly in recent decades. One of the problems that intrigues mould design engineers is how an injection mould can be designed in the most cost-effective way. A number of factors have to be considered in the mould design process. However, the first task in the mould design process is the determination of main parting direction, side-core direction and split core direction. These fundamental elements govern the manufacturing process, fabrication cost and the working life of the mould as well as the design lead time.

In practice, mould design is done manually and therefore, the design quality depends on the experience of the mould designer. Automating the mould design process requires an algorithm to resolve undercuts and to determine the parting direction as the first step in the automated process. In this research, algorithms for the determination of side core direction, split core direction and main parting direction are implemented. This provides a consistent and systematic method for analyzing and suggesting feasible parting directions for injection moulded parts.

In the proposed approach, geometric information of undercut feature for the possible parting direction is extracted first. This is attained through the use of heuristic search techniques, visibility maps and solid sweep. The algorithm is based on the geometric considerations. Other factors such as

mould flow characteristic and aesthetics are not considered. Basically, the proposed algorithm is composed of three main modules : determination of main parting direction for 2-piece mould, determination of side core direction, and determination of split core direction. An experimental system was implemented and is integrated to an existing CAD system (CADDS5 from ComputerVision).

The user is only required to input an object through CADDS5. The system then generates the main parting direction, side core and split core directions, and hence determines the mouldability of a component.

ACKNOWLEDGEMENTS

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CONTENT

CHAPTER 1

Introduction

1.1. RESEARCH OBJECTIVE

The injection moulding process is known to be superior for mass production of intricate parts at low production cost. However, the design of injection mould is a highly skill-intensive activity and relies heavily on human expertise. The quality of the designs thus largely depends on the designer's experience. In the manual design process, error or mistake is unavoidable. Hence, extra time is required for corrective tasks. In order to reduce the lead time of the design process and to minimize human errors, integration of computers in the mould design process to perform the deterministic and analytical tasks in some stages provides an effective solution. This research mainly concentrates in the initial stage of the mould design process - determination of main parting direction, side-core direction and split core direction.

1.2. THESIS ORGANISATION

This thesis is organised as follows : The necessary background information associated with injection mould design is reviewed in Chapter 2. This includes an introduction to the terminology and fundamental structure of a mould. Some general guidelines for mould design are also described.

Chapter 3 describes previous works related to computer-assisted mould design. An overview of the proposed algorithm is also presented.

Chapter 4 outlines the conditions of mouldability of a part with simple 2-piece mould, side cores and split cores, and it also gives a detailed discussion on the application and importance of solid sweep, Spherical map and Visibility map in the current research.

Chapters 5, 6 and 7 give detailed descriptions on the three main parts of the experimental system - determination of main parting direction for simple two-piece mould, determination of main parting and side core directions, and determination of split core directions.

Chapter 8 describes a depth-first search strategy for the solution.

Chapter 9 outlines the implementation of the experimental system. The search strategies are tested for a number of different cases. Analysis of the test results are discussed.

Chapter 10 analyses the complexity of the system.

Chapter 11 concludes this research and suggests some further works related to this research.

CHAPTER 2

Background on Mould Design

One of the most common and versatile moulding processes is injection moulding. Advantages of injection moulding include rapid replication of parts with complex geometries and low mass production cost. However, there are many parameters affecting the quality of a moulded part such as mould structure and moulding conditions. Until now, a rigorous analytical formulations for mould design is not available. Trade-offs are usually required in mould design such as the location of cooling ducts and ejection pins, and the location of parting line and side core slides. Since the design of injection mould relies heavily on human skill and intelligence, the quality of mould design depends mainly on the designer's experience. In order to reduce the lead time of the design process and minimize human error in mould design, integration of computer for modeling the interactions of the parameters in the mould design process will definitely be advantageous. Determination of parting direction, side-core direction and split core direction is the first step in the mould design process. This is essential for automating the mould design process. This research concentrates on the geometric aspect of the moulded part in the selection of parting direction, side-core direction and split core direction.

2.1. The Mould Design Process

In the conventional mould design process, the output is a complete set of engineering drawings showing the detailed structure of a mould. The process starts by considering the number of parts to be produced in one mould in one shot. This is followed by the arrangements of the cores and cavities in the mould base. In this stage, factors to be considered involve the ease of demoulding, the mechanism to be used for clearing undercuts, and the connections between mouldings and runners, and their effects on the part quality.

In addition, it is necessary to take into account the mould plate thickness and the ejector system which has to be capable of withstanding the loading on the mould. This includes the clamping pressure, the internal pressure when the mould is being filled and also the forces necessary for demoulding. Finally, the placement of cooling channel is essential for obtaining high quality mouldings with the mould operating economically.

Figure 2.1 shows a typical mould design process. The arrangement of cores and cavities in the mould base is mainly affected by parameters that are solely dependent on the geometry of the moulded part. They are the parting direction and parting line.

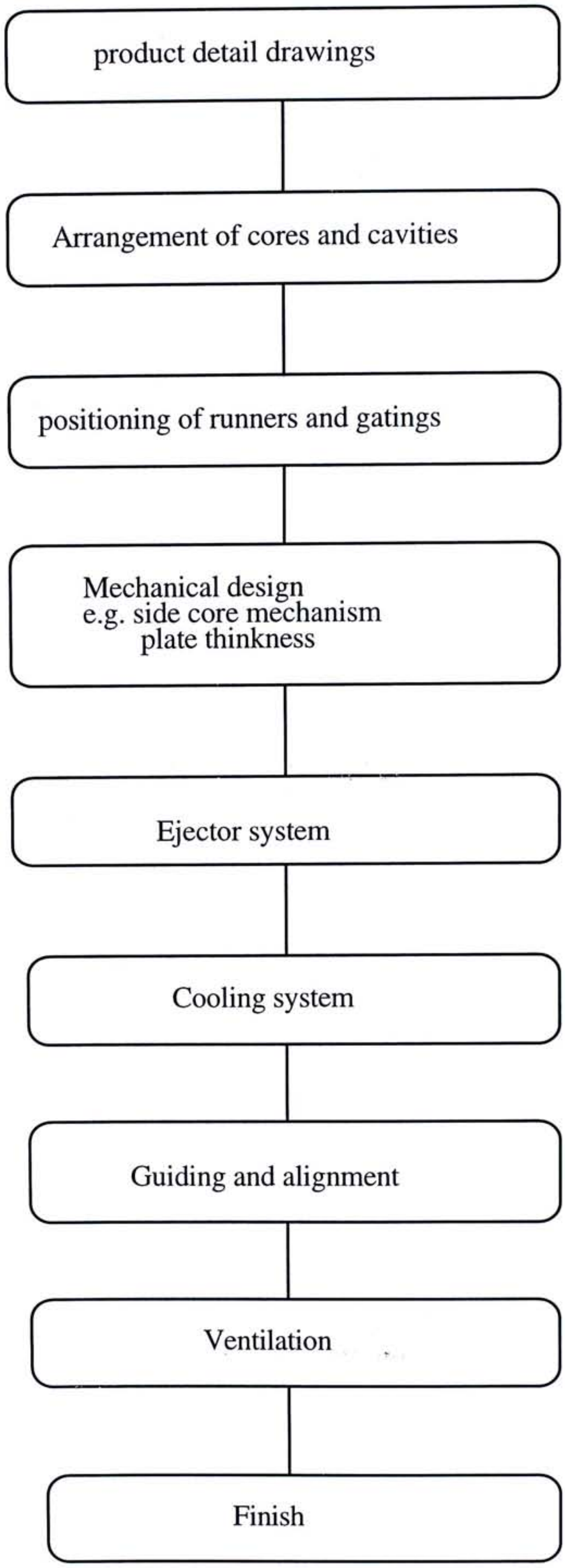


Figure 2.1. A typical mould design process

2.2. Basic structure of a two-piece mould

The injection mould is an assembly of parts containing an impression into which plastic material is injected and cooled. The cooled plastic material is ejected away from the mould and becomes the required product. Two-piece mould is a basic and simple type of mould. In a two-piece mould, the impression is formed by two plates - cavity plate and core plate [ref. 1].

The cavity plate, which is usually the female portion of the mould impression, gives the external profile of the moulding. Whereas, the core plate, which is usually the male portion of the mould impression, forms the internal shape of the moulding. In addition, the pair of opening directions of these two mould plates for ejecting the moulding away from the mould are called main parting direction (Figure 2.2).

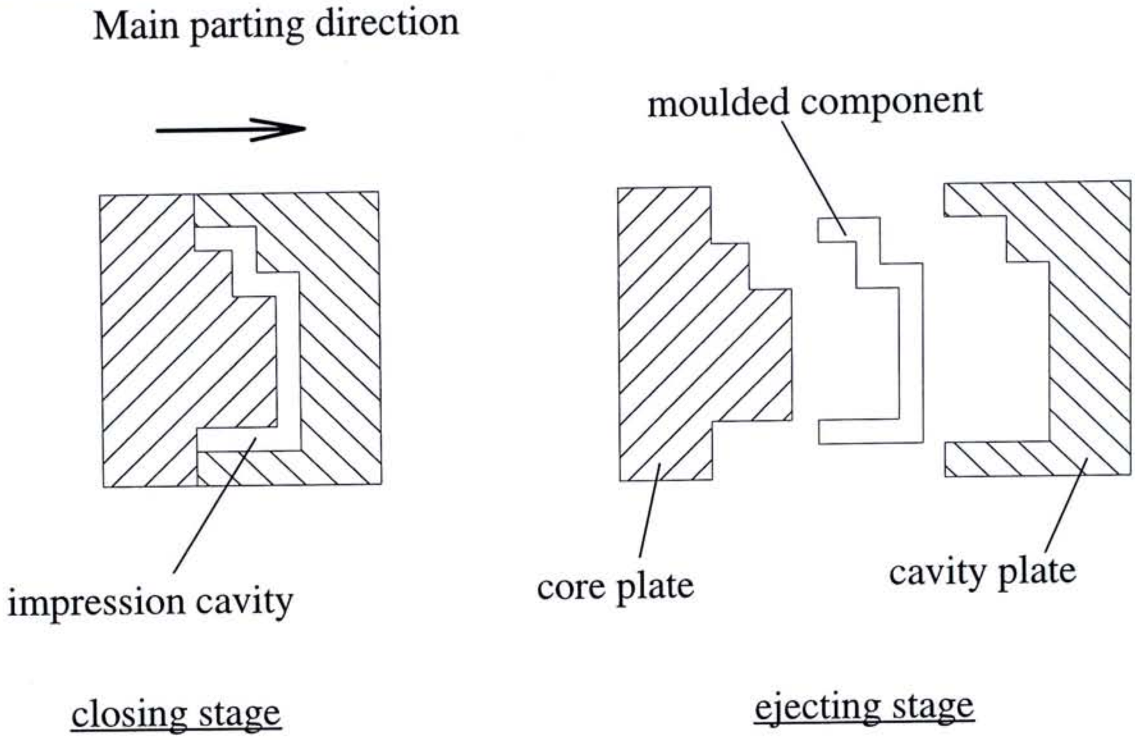


Figure 2.2. Structure of a simple two-piece mould

2.3. Undercuts

Frequently, the mould designer is unavoidably confronted with a component consisting of a recess or projection which prevents the removal of the moulded parts from the mould. These recesses or projections of a moulded part are called undercuts. It is important to minimize or eliminate undercuts in a moulded part as undercuts may lead to shorter mould life, difficulties in maintenance and higher tooling cost. To minimize the undercuts and reduce the number of side core, two most straightforward methods are usually used. One is to modify the structural design of the moulded part and the other is to modify the mould structure by adding side core or split core. In general, undercut can be classified as either internal or external.

2.3.1. External Undercut

External undercut is an undercut formed on the outside surface of a component. Examples of component which incorporate external undercuts are shown below (Figure 2.3).

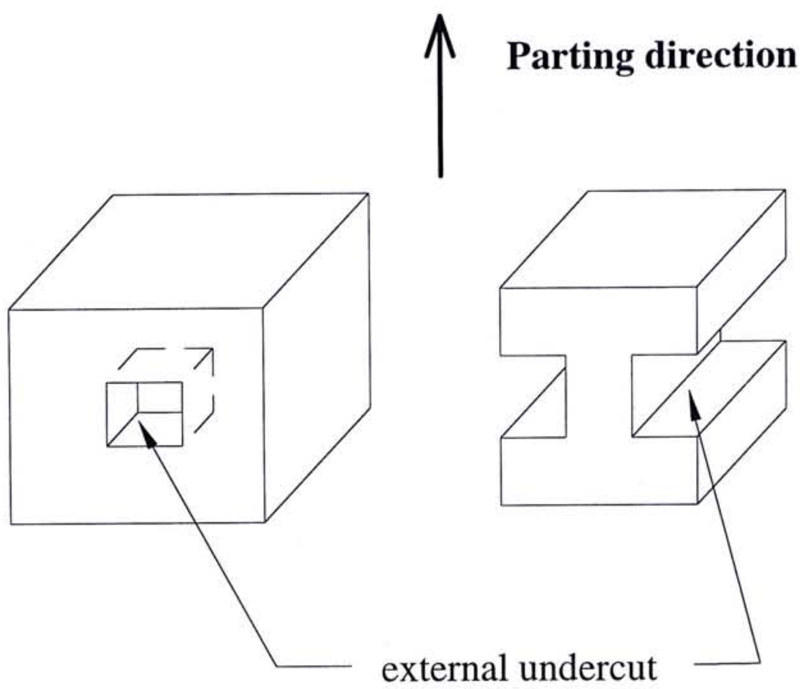


Figure 2.3. Examples of external undercut

To mould a part having external undercuts, two approaches are commonly used in practice. One is to consider different orientations of the parts with respect to the main parting direction, and the other is to add side core(s) (external slide) to the mould (Figure 2.4). The side core (external slide) is a movable metal block forming the external undercut of a moulded part. When the mould opens, the side core will have a guided motion away from the undercut. Obviously, mould with side cores involves the use of complicated mechanism. Higher tooling and maintenance costs are thus required.

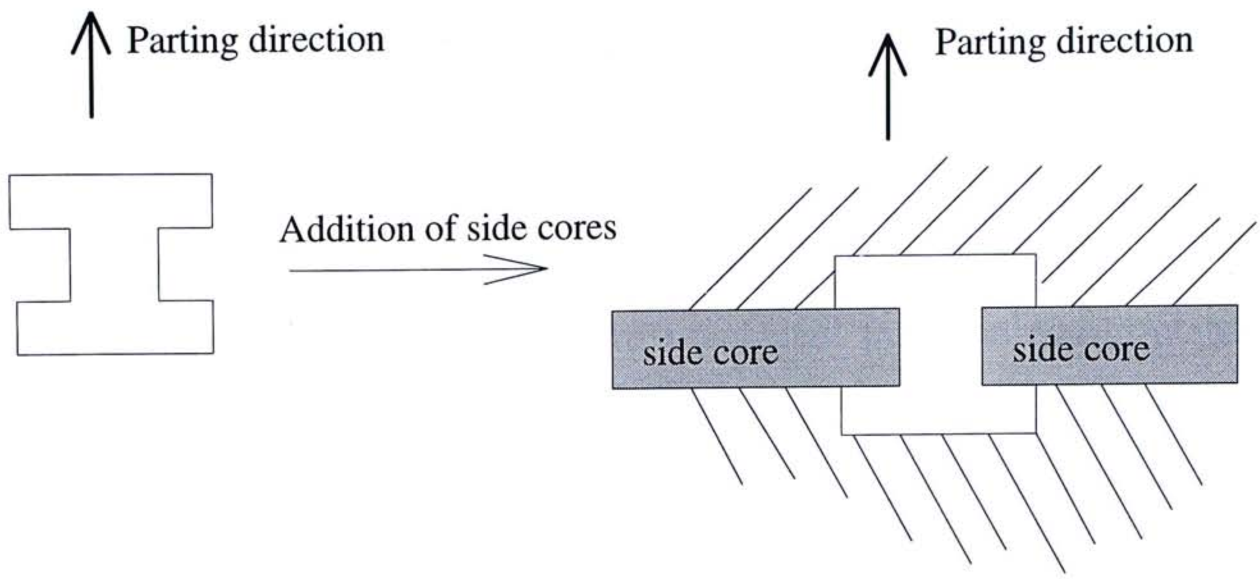


Figure 2.4. External undercut is resolved by side cores

However, the existence of external undercuts may vary depending on the main parting direction selected. Different main parting direction may result in different external undercuts (Figure 2.5). Thus, it is better to describe the external undercuts together with the main parting direction concerned. Based on this notion, a part having undercut in a parting direction, may be free of undercut in another parting direction. Hence, the other approach for resolving undercuts is to consider different orientations of the part such that no undercut is resulted.

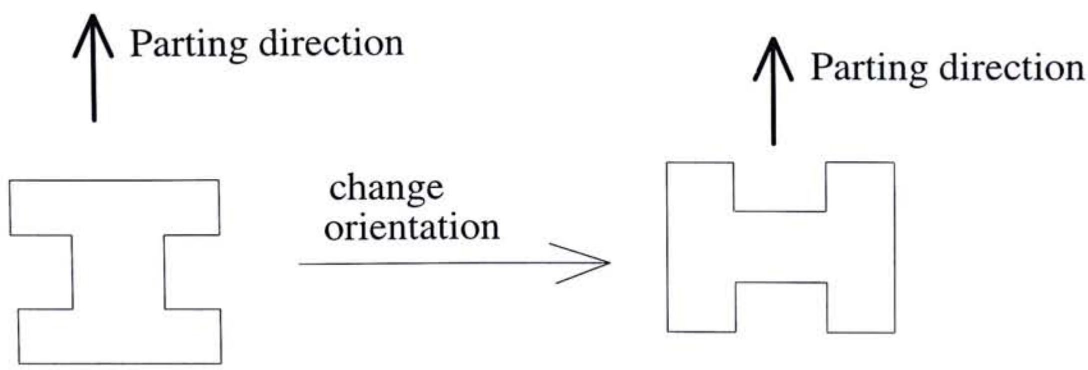


Figure 2.5. No undercut is resulted by changing the different orientations of the part

2.3.2. Internal Undercut

Internal undercut is an undercut on the internal wall of a part (Figure 2.6) preventing the moulded part to be ejected from the mould plate. In practice, an internal undercut is resolved by using a split core mechanism as shown in Figure 2.7.

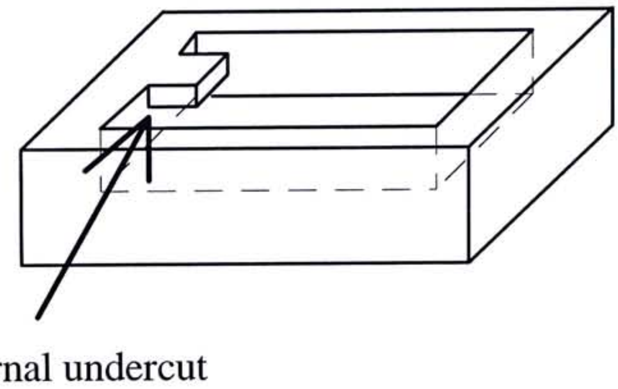


Figure 2.6. Example of internal undercut

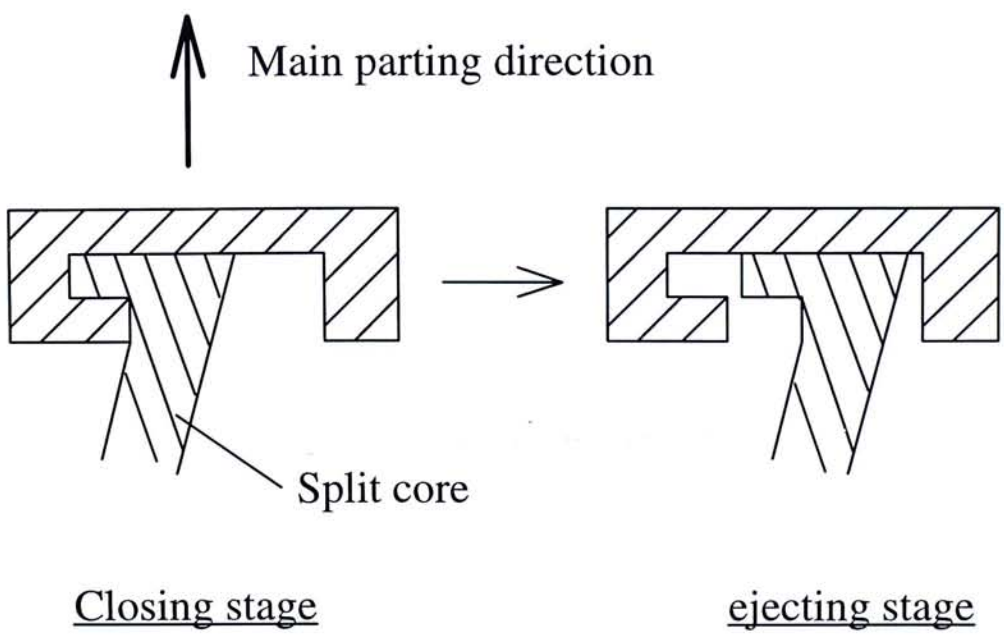


Figure 2.7. Internal undercut is resolved by split core

When the mould is opened, the split core is guided to move away from the undercut. Thus, it is important to ensure that there is no blocking in the trajectory of split core motion.

CHAPTER 3

Related works

3.1. Previous works

There are few published articles related to this area. K.C. Hui and S.T. Tan [ref. 2] proposed to use the projected area and the degree of blocking for determining the main parting direction. The degree of blocking of external undercut of an object along a particular main parting direction is measured by generating a series of test points on the edges of the object. Then, by applying a method which is similar to ray-casting but with the rays emerged from the test points, a number of rays are generated and are then intersected with the object. As a result, the degree of blocking of the undercuts for each candidate parting direction can be evaluated. On the whole, the approach can only apply to objects without internal undercut.

B. Ravi and M.N. Srinivasan [ref. 3] proposed a decision support tool to assist user to select an optimum parting direction. User is required to input a main parting plane and the suitability of this parting plane is then evaluated. The determination of directions for side core and split core is not considered.

M.A. Ganter and L.L Tuss [ref. 4] provided another view to the problem of parting line determination. Three types of parting line search methods for parting line are developed, search along a user specified direction, search about the object's center of gravity, and search along the object's principle axis. All of these three search operations are based on test performed on a set of randomly generated parting planes. This approach may require processing a large number of test planes before a proper parting plane is determined. Furthermore, the optimum parting plane cannot be determined easily as the test planes are generated randomly independent of the object geometry.

Recently, L.L. Chen, S.Y. Chou and T.C. Woo [ref. 5] suggested the application of visibility mapping in mould and die design. In spite of its usefulness and efficiency in determining parting direction, internal undercut is unresolved.

3.2. Overview of the proposed approach

This research project mainly focus on the initial mould design stage, determination of main parting direction, side core direction and split core direction. It is the first step in an attempt to automate the mould design process. A fundamental problem in mould design is the lack of geometric information of undercut feature which is not explicitly provided by a part description so that extraction and recognition of undercut feature are required. Two types of undercut are required to be distinguished, internal and external undercuts which have to be resolved by different mechanisms. In addition, the classification of an undercut feature for either catagory is not dependent on the local geometric information of the feature. The essential factors for the classification are the orientation of the part and the blockage of the side core movement. In general, if the undercut cannot be resolved by any external side core, then it is classified as an internal undercut. Based on this notion, a method is developed for resolving undercuts. The method is composed of three main parts,

- Determination of main parting direction for 2-piece mould.
- Determination of side core.
- Determination of split core.

The structure of the algorithm is shown below (Figure 3.1).

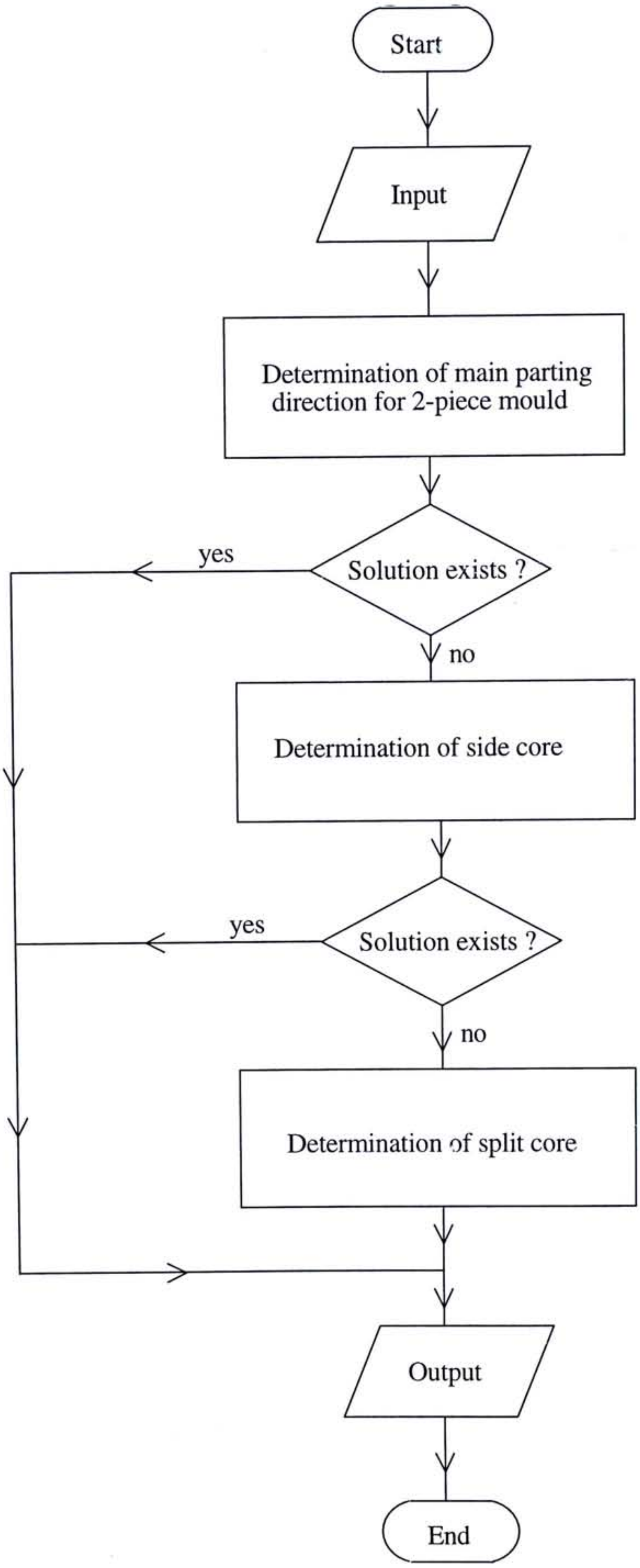


Figure 3.1. Structure of the proposed system

The first part of the system is used to determine the main parting direction for a 2-piece mould. If all undercuts are resolved in this part, this object can be moulded by a simple 2-piece mould. Otherwise, the next part is to try resolving all undercuts with side cores. If all remaining undercuts can be moulded by side core, no further processing is required and the selected directions are returned. Finally, unforunately, if there is undercut still remaining after consideration of side core, this remaining undercut will be considered as internal undercut. The last module is then invoked to resolve this internal undercut. If undercut still exists after the consideration of split core, the object is considered as unmouldable.

CHAPTER 4

Background Theories

4.1. Mouldability of a part

4.1.1. Mouldability of a part by simple 2-piece mould

The condition for mouldability of a part can be treated as a visibility problem. Note that a main parting direction is composed of two opposite but parallel directions. If any points on the part can be completely viewed along any one of this pair of directions, then this part can be moulded directly by a simple 2-piece mould and this direction is the required main parting direction. Otherwise, the part cannot be moulded by a simple 2-piece mould.

4.1.2. Side Core for Mouldability of a Part

For the case of part having external undercut, the condition for mouldability is just an extension of the case for simple 2-piece mould. In the existence of external undercut, unresolved undercut feature will remain if a simple 2-piece mould is used. Addition of side core is necessary for resolving the remaining undercut. If any points on the surface of the unresolved undercut can be seen along the side core direction, then this part is regarded as mouldable with the addition of side core in its mould.

4.1.3. Split core for mouldability of a part

In the existence of internal undercut, there is at least one face of the part which is not visible completely or partially along any directions. In this situation, simple visibility test is insufficient and other testing method is required. If the trajectory of the split core is free of interference, this split core is feasible for resolving an internal undercut. Thus, a straightforward approach for testing the feasibility of a split core direction is to detect the existence of any interference along the trajectory of split core motion.

4.2. Solid Sweep

Solid sweep and visibility map are used for detecting the feasibility of a testing direction. The sweep approach is based on the notion of moving a point, curve or surface along a certain trajectory. The locus of points generated or the volume being swept out defines a one, two or three dimensional object. As in Figure 4.1, a solid is generated by translational sweeping of the surface S along the vector V which is non-coplanar with surface S.

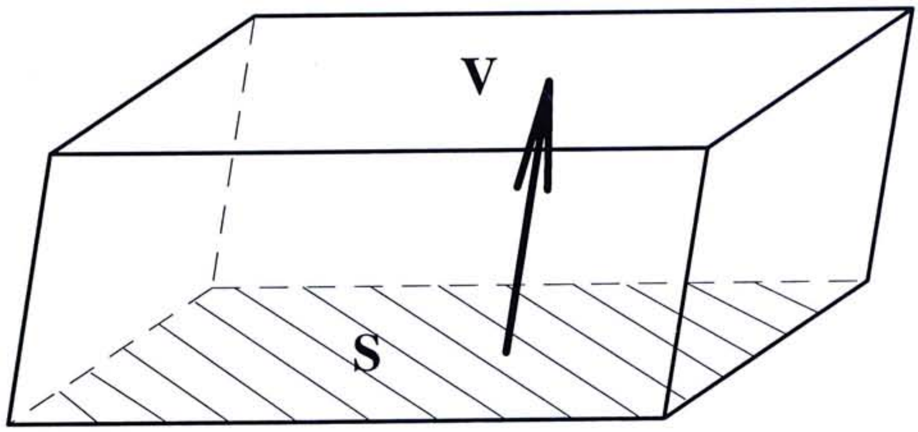


Figure 4.1. Solid formed by sweeping of the surface S

Consider a polyhedral solid composed of a number of planar surfaces. A solid sweep consists of two steps. Firstly, a sweep operation is applied to each of the surfaces of the solid. Secondly, the result of the sweep operation are united to obtain the final swept solid. However, in a surface sweep operation, dangling surface (Figure 4.2) may exist if the sweep direction is coplanar with the candidate surface.

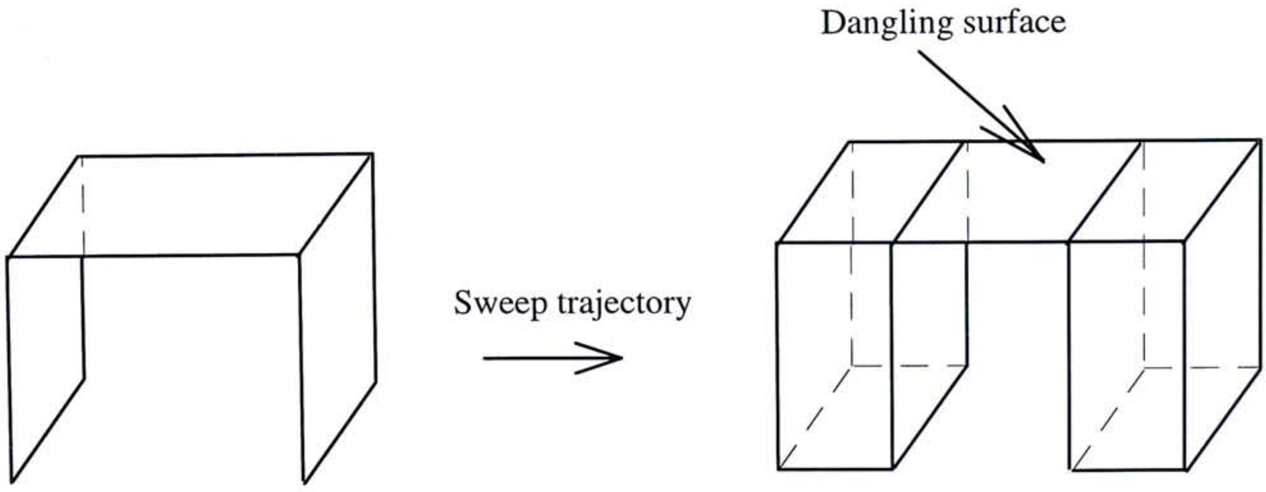


Figure 4.2. Formation of dangling surface

To avoid the existence of the dangling surface in the solid sweep operation, the vector dot product between the unit normal vectors of each surface and sweep direction (Figure 4.3) is evaluated before applyng the sweep operation on each surface. If the result of the dot product is zero, the surface is not considered for the subsequent sweep operation.

i.e. If $\vec{N} \bullet \vec{S} = 0$, then no sweeping operation is performed.

where $\vec{N}$ = unit vector of the surface normal

$\vec{S}$ = unit vector of the sweep direction

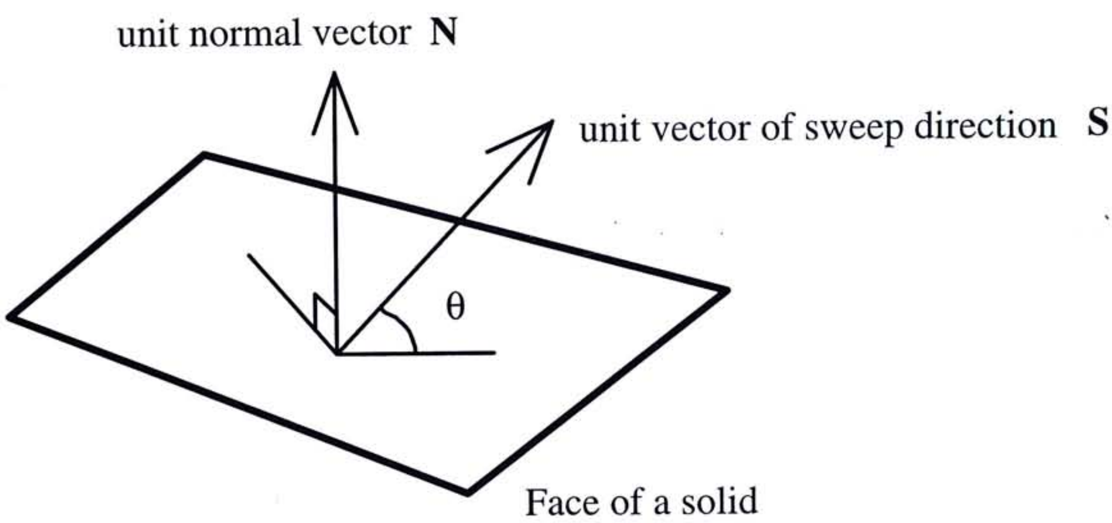


Figure 4.3. Validity of sweeping operation

A number of swept solids are then obtained. A union operation is then performed on these swept solids to obtain the region generated by sweeping of the original solid (Figure 4.4).

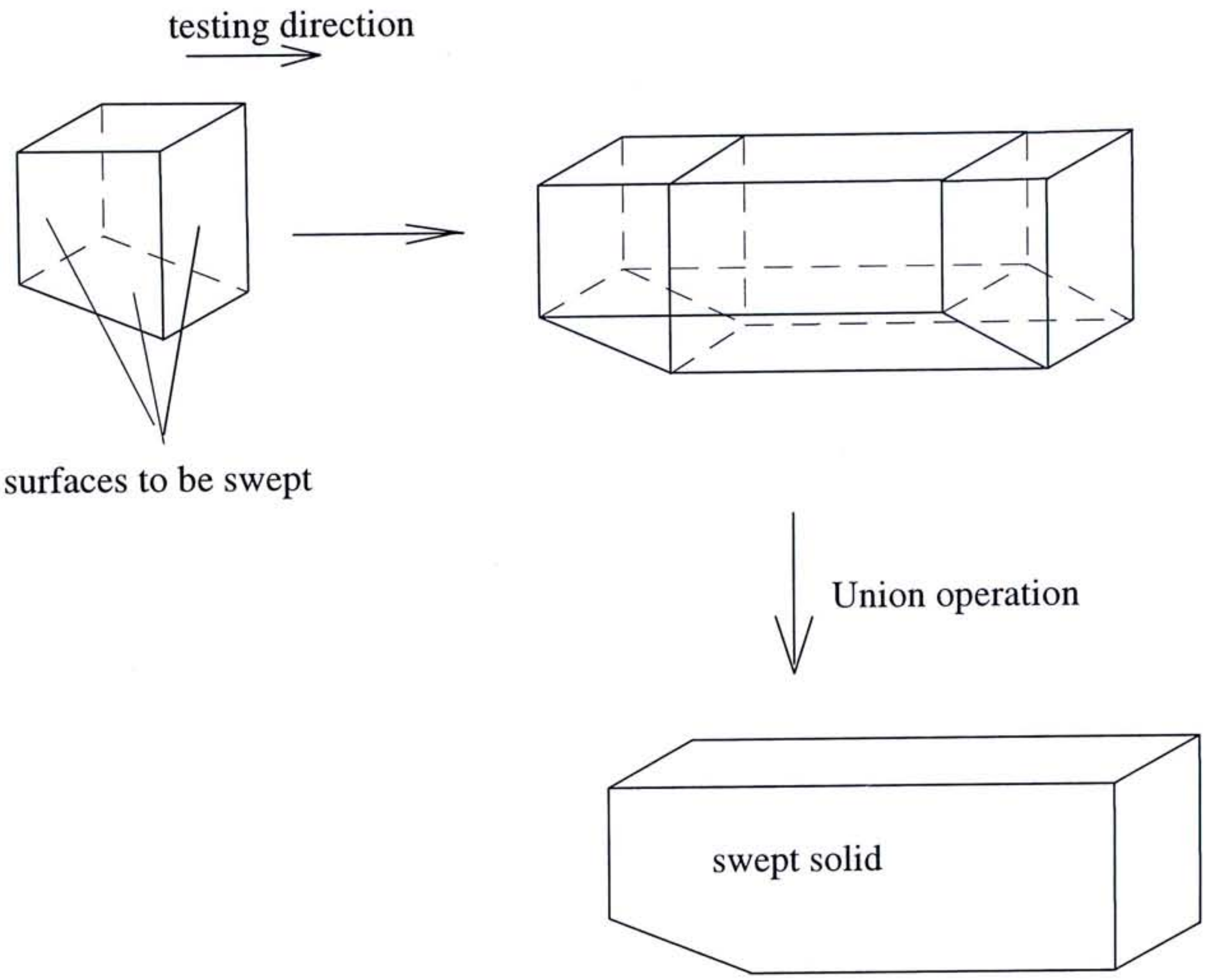


Figure 4.4. Example of solid sweep

4.3. Application of Solid Sweep in Mould Design

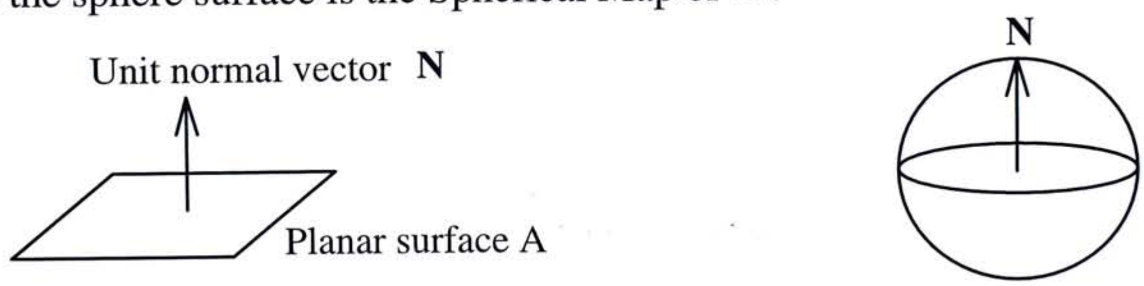
The necessary condition for the design of side core or split core is that there have to be no blocking in their motion. Thus, it is important to detect the existence of interference in their trajectories. This is attained by detecting the interference (with boolean operation) between the original part and the region swept out by the solid sweep. If there is no interference, this direction is a feasible side core or split core direction.

4.4. Spherical and Visibility Mapping

4.4.1. Spherical Mapping

Another method for deciding the feasibility of a direction is the use of Visibility Mapping. Spherical Mapping is an intermediate step for the construction of a Visibility Map which will be discussed first.

The Spherical Map (also known as Gauss map) [ref. 5 and 6] is defined as a spherical representation of a surface by translating the unit normals of a surface onto unit sphere. The Spherical map has many applications such as cutting-tool orientations [ref. 7] and surface offsets [ref. 8] for NC machining, and it also assists in shape reconstruction from imaging [ref. 9] in computer vision. In Figure 4.5, a plane surface with its surface normal is shown. The Spherical Map is constructed by translating the unit normal vector $\vec{N}$ into the unit sphere. The intersection point of $\vec{N}$ and the sphere surface is the Spherical Map of $\vec{N}$.



Planar surface A with its unit normal vector — Spherical map for the planar surface A

Figure 4.5. Concept of Spherical Mapping

Figure 4.6 shows examples for the formation of a Spherical Map. For the case of a cylindrical surface, the Spherical Map obtained is a full circle at the equator of the unit sphere.

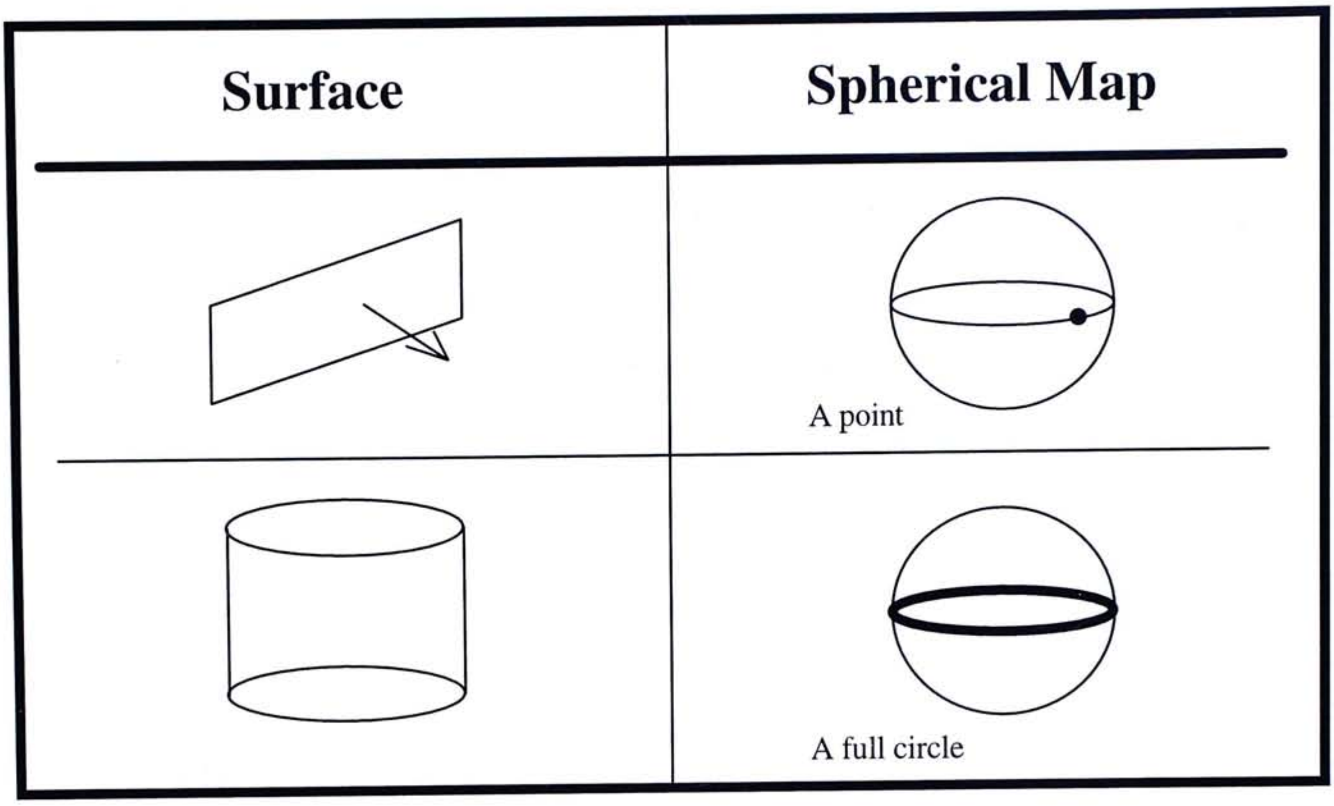


Figure 4.6 . Examples of Spherical Map

4.4.2. Visibility Mapping

The Visiblity Map [ref. 5] (Vmap) is a set of unit vectors on an unit sphere such that each of which differs from the normal of the surface by at most 90°. A Vmap is a spherical region similar to a Spherical Map except that a point in a Vmap denotes a direction such that the entire surface S is visible to its exterior. The normal at a point on a surface gives the direction in which the point is visible from infinity if the ray encounters no intersection. However, the same point can be visible from many other directions, up to a hemisphere of directions bounded by the tangent plane at that point.

Since a Spherical Map records normal direction of a surface, a visibility map can be constructed from a spherical map. Figure 4.7 illustrates the construction process of the visibility map from the Spherical map. Figure 4.7a shows a Visibility map produced from a plane surface. The unit normal vector of the surface is first translated to the unit sphere. A hemispherical shell is then constructed shown by the shaded portion. For objects with more than one surface, the visibility map is the intersection region of the hemispherical shells of individual planes. The Visibility Map of an object with two faces is shown in Figure 4.7b.

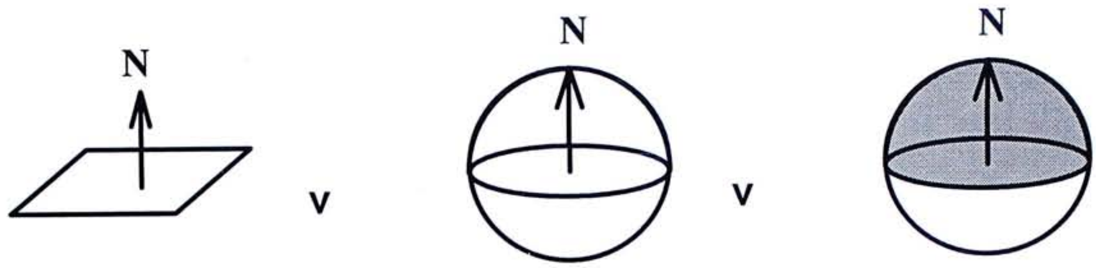
Procedure V_map(Spherical_map)

V_map $\leftarrow$ entire sphere

For each point p_i on Spherical_map

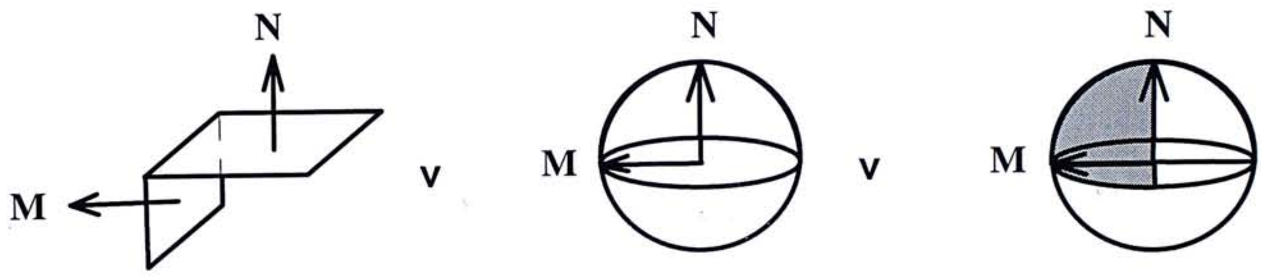
    V_map $\leftarrow$ V_map $\cap$ hemisphere(p_i)

End V_map



A surface with its unit normal vector **N** Spherical map Visibility map

Figure 4.7a



Two faces with their unit vectors **M** and **N** Spherical Map Visibility map

Figure 4.7b

Figure. 4.7. Construction of Visibility Maps

For polyhedral objects, if all faces of an undercut feature are visible along a certain direction, this direction is a valid direction for resolving the undercut. An undercut is thus said to be mouldable along a direction if it is completely visible along that direction. In this research, Visibility Map is mainly applied for testing the feasibility of a direction in resolving undercut.

Given a point p on the unit sphere formed by a direction **d** and the Visibility Map $V(f_i)$ of each face f_i of an object, if the undercut represented by the faces f_i is mouldable along the direction **d**, then the following conditions are satisfied.

Condition 1 : $V(f_1) \cap V(f_2) \cap \ldots\ldots \cap V(f_m) \neq \phi$

where m = no. of faces of the undercut

Condition 2 : $p \in V(f_1) \cap V(f_2) \cap \ldots\ldots \cap V(f_m)$

Condition 1 shows the necessary condition for the mouldability of a part. A visible zone in the Visibility Map must exist if the the part is regarded as a mouldable one. Actually, the visibility zone can be formulated as

$$\text{Visibility zone} = V(f_1) \cap V(f_2) \cap \ldots\ldots \cap V(f_m)$$

where m = no. of face of the undercut

If condition 1 is not satisfied, there is no intersection among the hemispherical images so that no direction exists along which all faces are visible. Hence, the corresponding undercut cannot be resolved.

In addition, Condition 2 implies that if p lies within the visible zone of the Visibility Map, then **d** is a feasible direction for resolving the undercut. Thus, if condition 2 is not satisfied, the direction in consideration cannot be used for resolving the undercut.

Hence, if the direction is a valid direction for resolving undercut, both conditions must be satisfied.

CHAPTER 5

Determination of Main Parting Direction for 2-piece Mould

A 2-piece mould is a simplest kind of mould. Due to its simplicity, 2-piece mould is usually preferred in mould design. In this case, it is only necessary to determine the main parting direction. Two factors are considered in the determination of main parting direction. They are the projected area of the moulded part and the degree of blocking. Ideally, a main parting direction having largest projected area and free of undercut is preferred. Based on this criteria, possible directions are selected and tested. The most appropriate one can then be determined. Figure 5.1 shows an overview of this module.

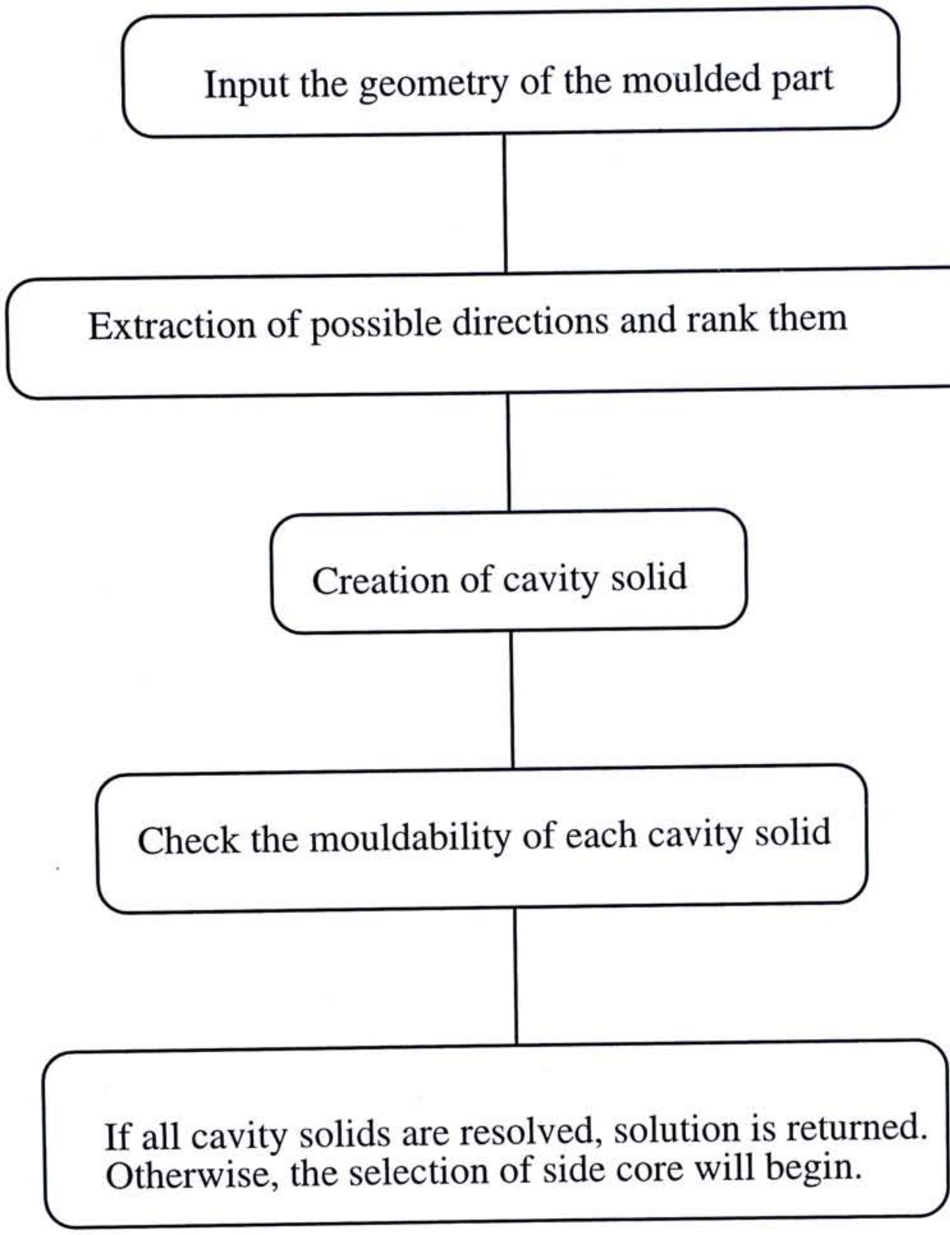


Figure 5.1. Overview of determination of main parting direction for 2-piece mould.

5.1. Extraction of possible main parting directions

The set of all possible directions is constructed by extracting all face normals. This is because a moulded part is usually oriented to have a surface on or parallel to the parting surface for ease of machining (Figure 5.2). In orientation 1 in Figure 5.2, only one mould plate is required for machining so there is no problem in alignment between the two mould plates. In contrast, in orientation 2 (Figure 5.2), not only there is a difficulty in machining, but the two mould plates also require very good alignment. Slight misalignment of the mould plates will seriously affect the product appearance and its functional performance. Thus, all face normals of a part is extracted as the main parting direction for polyhedral parts.

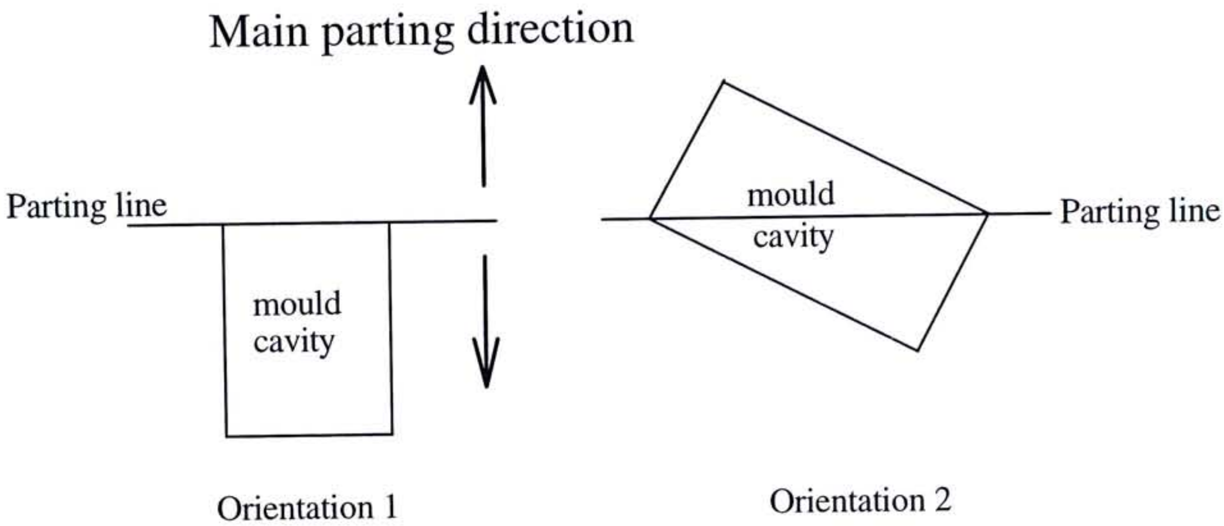


Figure 5.2. Example of face normal of part to be parting direction

5.2. Main Parting Direction

In mould design process, the main parting direction is usually taken to be the direction with minimum number of side cores or split cores. It is thus desirable to reduce the number of side cores or split cores whenever possible since the use of them increases tooling and maintenance costs significantly.

As a result, a practical rule for selecting main parting direction is to select the one with no or minimum undercut since the manufacturing cost can be reduced. In addition, the projected area of the part along a direction may also affect the choice of the main parting direction. In general, the area in shear contact between the mould plate and the moulded part should be kept to a minimum. This is essential for reducing possible distortion which may be induced on the moulded part during the mould opening operation. In order to minimize the area in shear contact, a main parting direction with a maximum projected area of the moulded part is usually preferred. Ideally, a main parting direction is preferred to satisfy these two conditions. However, this is not always possible and a compromise is always necessary.

5.3. Ranking of Main Parting directions

In the selection of main parting direction, a direction is selected from the set of directions for blocking test. In order to reduce the time for the search of main parting direction, it is essential to rank the parting directions in an order such that the most appropriate one has the highest priority to be selected. The ranking is based on the projected area of the moulded part along a direction. The larger is the projected area along a direction, the higher is the priority of this direction to be selected for test.

5.4. Calculation of projected area of a moulded part

The projected area of a part plays an important role in the determination of main parting direction. It is an usual practice to choose a parting direction with the largest possible projected area since the area in shear contact between the mould plate and the moulded part can be kept to a minimum. In addition, large lateral projected area which is perpendicular to the main parting direction requires a thicker side wall of the mould such that the size of mould plates are increased significantly. As a result, the mould base and other related components have to be changed to adapt such design. If the part is oriented to have the largest projected area along the main parting direction, it may be only necessary to increase the thickness of the mould plates because thin mould plate is not rigid enough against injection pressure and then distortion of mould plate will be resulted. However, adding supporting column for the mould plate is more preferable (Figure 5.3). The supporting column can strengthen the mould plate against the pressure. It can help to reduce the thickness of mould plate. Thus, it is always desirable to have larger projected area along the main parting direction. The projected area is a primary factor in mould design.

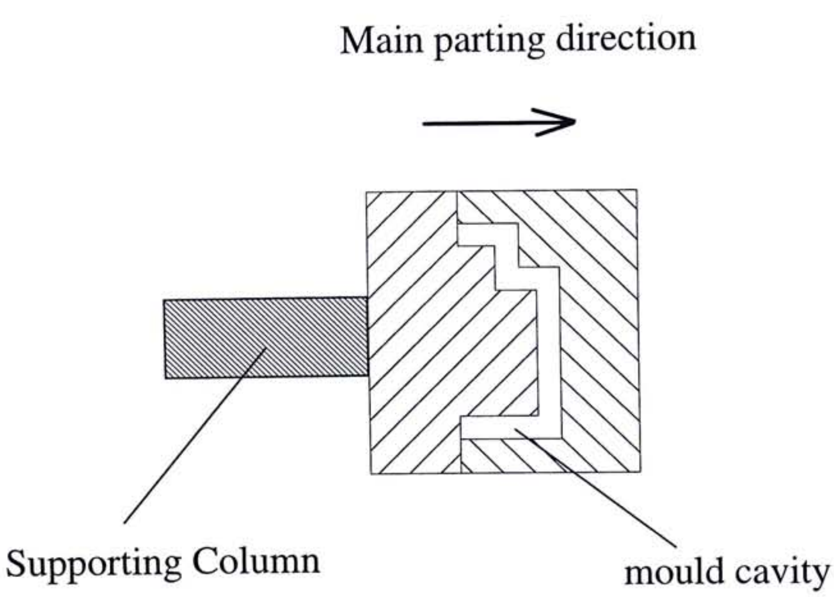


Figure 5.3. Example of a supporting column

A *Preference Value* is used to measure the projected area relative to the maximum area of an object for a particular parting direction. The preference value is defined as

$$\text{Preference value} = \frac{\text{Projected area of object along a given direction}}{\text{maximum projected area of object among the set of possible parting directions}}$$

Hence, a preference value must lie in the range 0.0 to 1.0. The higher is the preference value, the greater is the projected area and the more is the direction being preferred to be chosen as the solution.

However, a direct summation of the projected area of each face of the component in the given direction may not reflect the actual measure of the required area. As shown in Figure 5.4, direction D2 is a most favorable choice for parting direction although area A2 is smaller than A1. Thus, estimation of projected area of a moulded part takes the area enclosed by the convex hull of the projected profile of a part.

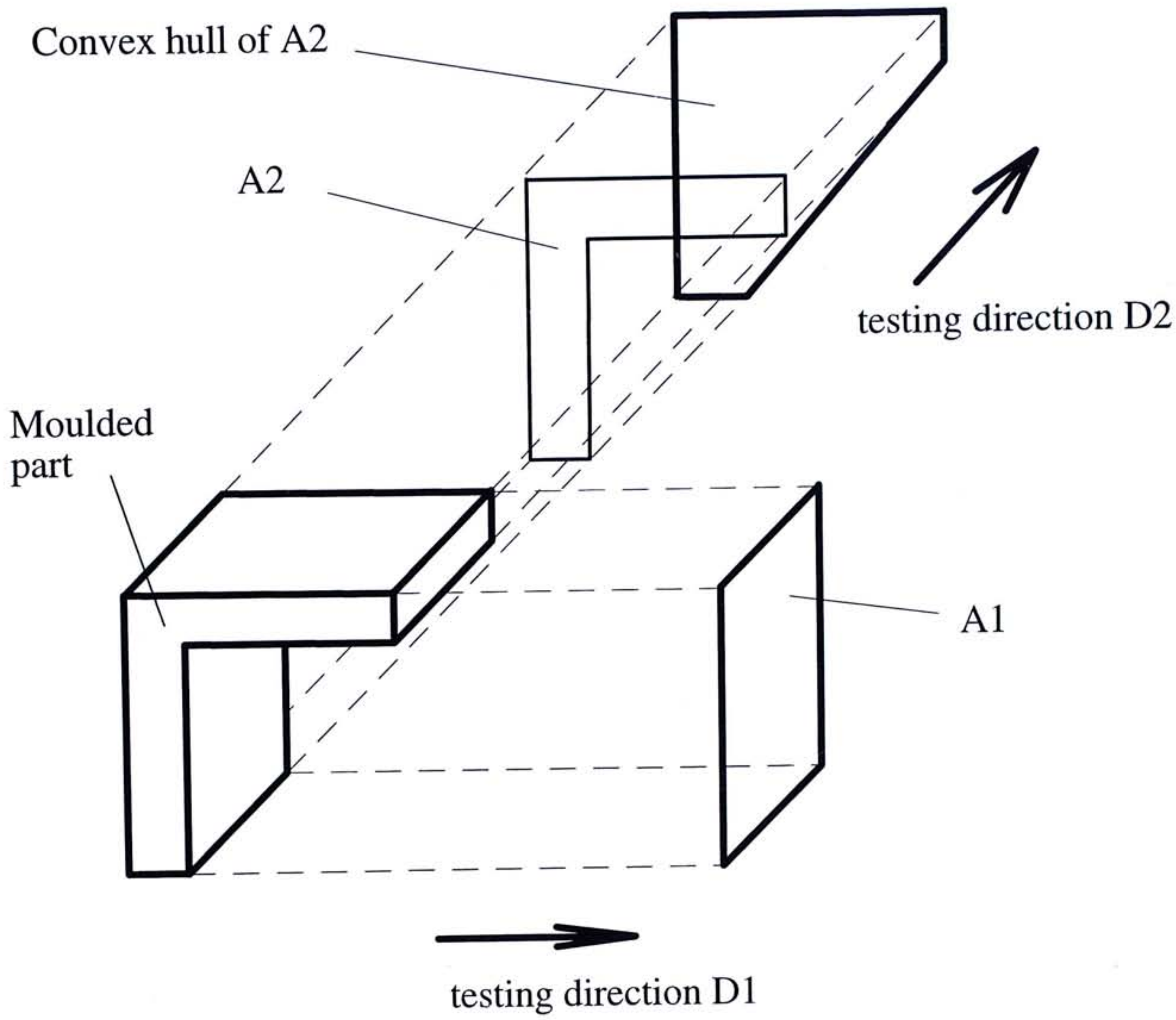


Figure 5.4. Projected area calculated from the convex hull of the part where (convex hull of A2 > A1 > A2

To obtain the required convex hull, all vertices of the part are projected along the testing direction. A convex hull algorithm [ref. 10] is then applied over these points to obtain the corresponding minimum enclosing convex polygon. Based on the Preference Values, the possible directions can then be arranged according to the preference of a direction being chosen as the parting direction.

5.5. Creation of Cavity Solid

In order to resolve undercut in mould design, explicit knowledge on the geometry and location of the undercut is required. Given a solid model of an object X to be moulded, resolving undercut requires extracting the cavity solid which contains geometric information of undercut. The cavity solid is created by subtracting object X from the convex hull of X. (Figure 5.5).

i.e. CS_X = CH_X - X
where CS_X : Cavity solid of object X
CH_X : Convex hull of X

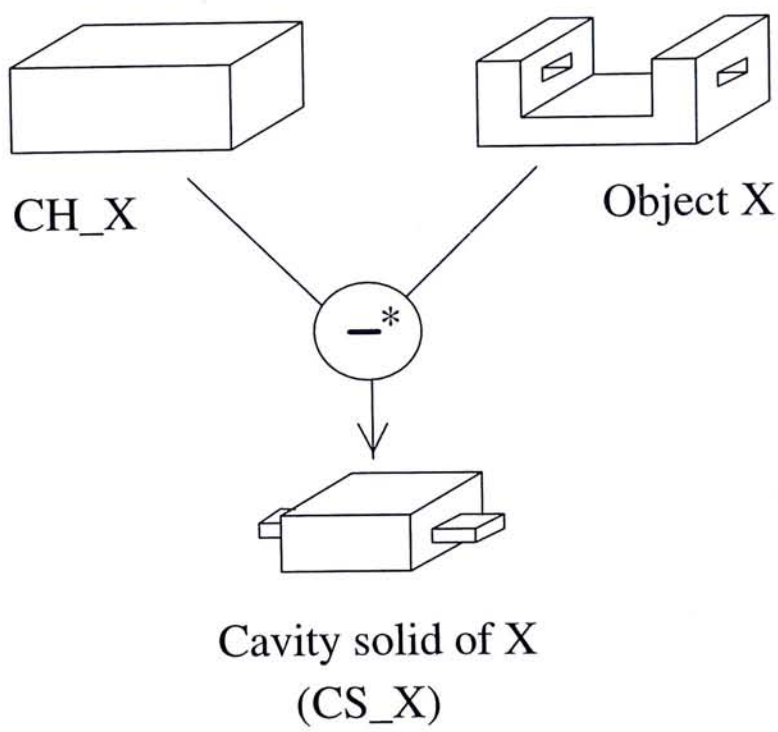


Figure 5.5. Creation of Cavity Solid

The gift-wrapping method [ref. 10] is adopted for constructing the 3-D convex hull of an object. Initially, the vertices of the object are extracted. Next, an initial plane of the convex hull is determined. The angles between this initial plane and all vertices on the edges of the plane are measured. Then, the vertices with largest angles are collected and the 2-D convex hull algorithm is applied to obtain a plane which is connected to one edge of the initial plane. This procedure is repeatedly used for all edges of the existing planes until all edges are shared exactly by two faces so that a closed convex solid is formed.

According to Figure 5.5, the geometry of the cavity solid is independent of the testing direction. In addition, since the cavity solid is formed by a regularized difference between the convex hull of the part and the part itself so the cavity solid must contain all undercut for all possible directions.

$$U \subseteq CS\_X$$

where CS_X : cavity solid of object X.

U : undercut solid resulted from any parting directions

Hence, the testing process can be confined in the cavity solid since it contains all possible undercut features. If all cavity solid can be resolved, the direction is a feasible parting direction. If part of the cavity solid is resolved, the remaining part must be the undercut solid for that main parting direction.

5.6. Cleavage of Cavity Solid

Direct application of visibility check to the cavity solid may not give the correct result. Figure 5.6 shows a typical example in which a part can be moulded along the indicated parting direction obviously but the Vmap and solid sweep approaches cannot give the same result. This is a result of the non-existence of a visible zone in the Visibility Map of the object in Figure 5.6, (Visibility Map of the cavity solid = $V(f_1) \cap V(f_2) \cap \ldots\ldots \cap V(f_{10}) = \varnothing$) since there is no direction so that all the ten faces are viewed. For the case of solid sweep, the object created by sweeping of the cavity solid along the parting direction interferes with the original part.

In order to eliminate this insufficiency, partitioning of a cavity solid along the testing direction before visibility test is conducted. This is attained by splitting the cavity solid with a series of cutting planes which are defined by the edges of cavity solid and the parting direction. However, only the edges of cavity solid which is non-parallel to the parting direction are selected. The selection of the edges requires to compute the dot product between the direction of the edge and the testing direction. If the dot product is not equal to ± 1, the edge is selected.

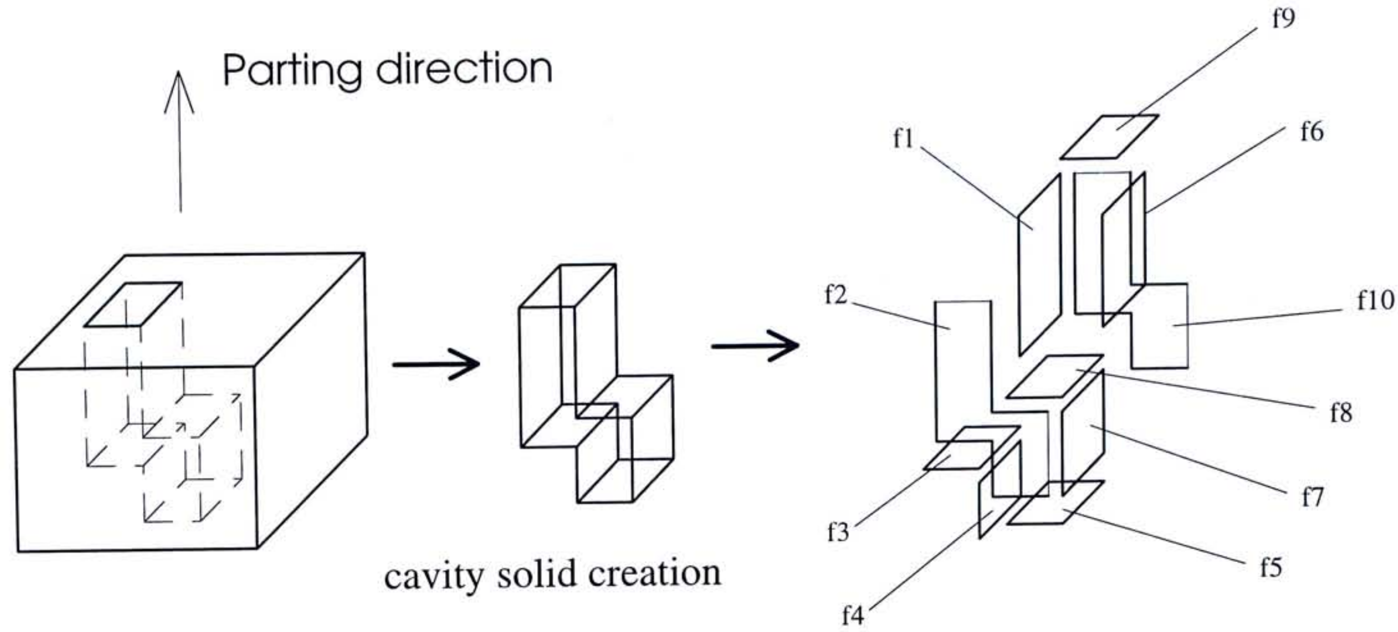


Figure 5.6. An example of part moulded along the given parting direction.

In Figure 5.7, the cavity solid is cut into two pieces by the cutting plane. Visibility check along the test direction is performed for each of the cavity solids. Obviously, the test result with solid sweep and Visibility Map are now consistent with the actual mould process. Thus, cleavage process for cavity solid is a pre-requisite for blocking check.

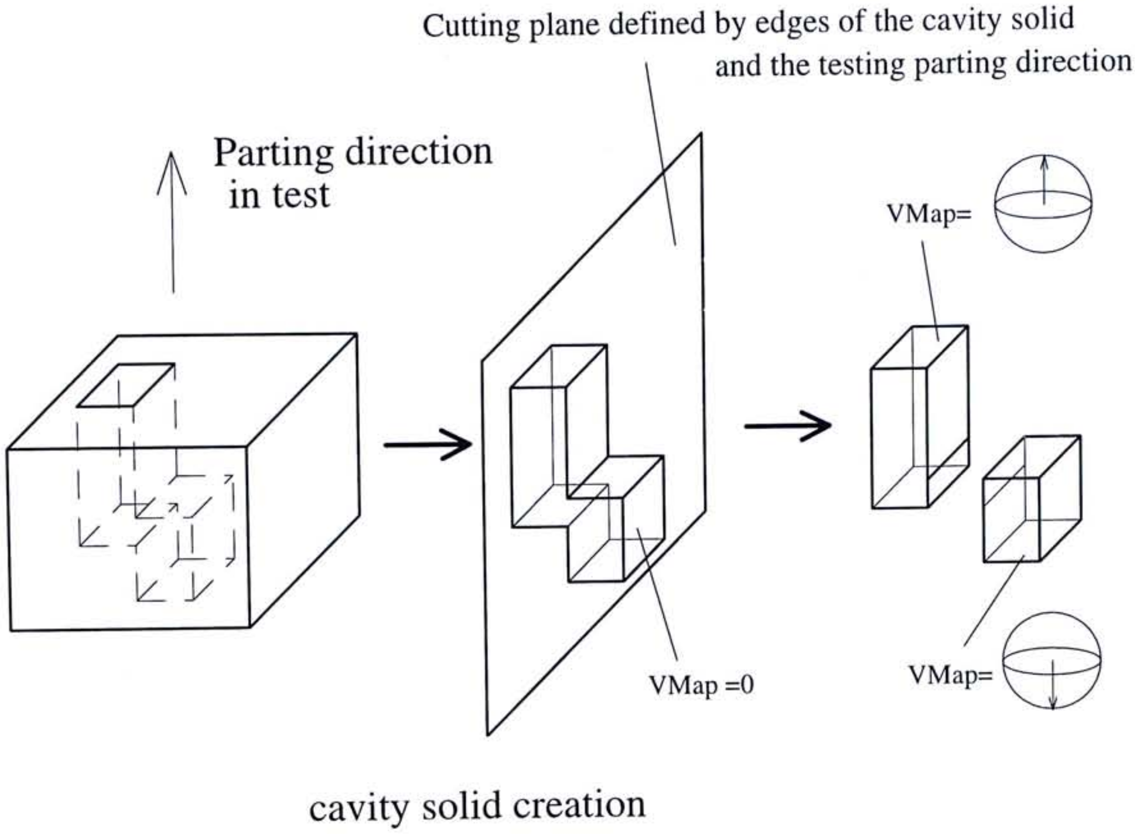


Figure 5.7. Cleavage of cavity solid

5.7. Undercut solid determination

As mentioned in previous sections, the cavity solid contains all possible undercuts of a solid. An undercut solid is thus part of the cavity solid that cannot be resolved along a given main parting direction. As shown in Figure 5.8, a cavity solid is separated into two smaller ones which are then used for visibility test in the given parting direction. Those solids not visible along the parting direction are the undercut solids with respect to the given parting direction.

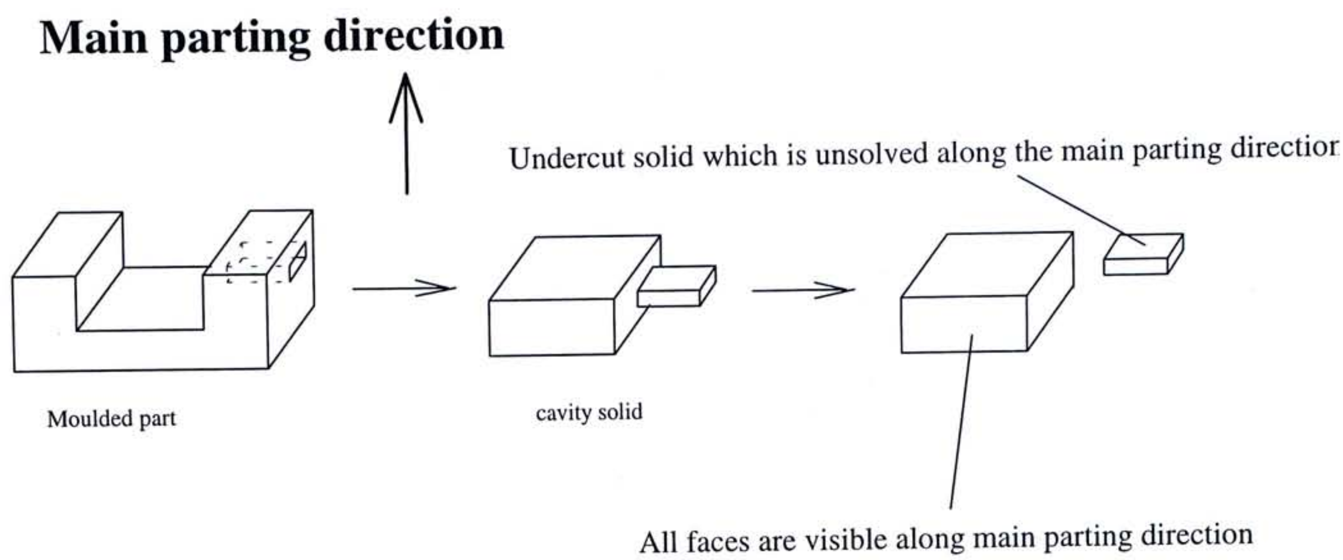


Figure 5.8. Example showing the steps for determining undercut solid

5.8. Difference in the Application Area of Solid Sweep and Visibility Map

As discussed in previous sections, both methods can be used for detecting blockage in a given direction. However, they have their own pros and cons. Therefore, the proposed approach tries to avoid their weaknesses by combining both methods to complement each other in the blockage test. This is attained through the use of a local interference and a global interference test. Local interference (Figure 5.9) is the blockage of the split core or side core induced by the cavity solid itself. Global interference is the interference induced by the moulded part. Visibility Map is sufficient for detecting local interference (Figure 5.9) of cavity solid while it is insufficient for detecting global interference (Figure 5.10). This is because Visibility Map only considers the geometry of the undercut solid while the moulded part is not taken into consideration. On the contrary, solid sweep is sufficient for detecting both local interference and global interference since it is a direct simulation of the split core and side core motion. However, solid sweep is computationally expensive comparing with Visibility Map. A compromise is thus to allow the Visibility Map to handle all local interference case while solid sweep is employed for global interference case. In order to reduce computation time, the frequency in the use of solid sweep is minimized by considering Visibility Map prior to the use of solid sweep. This requires classifying the cavity solid according to their types of interference.

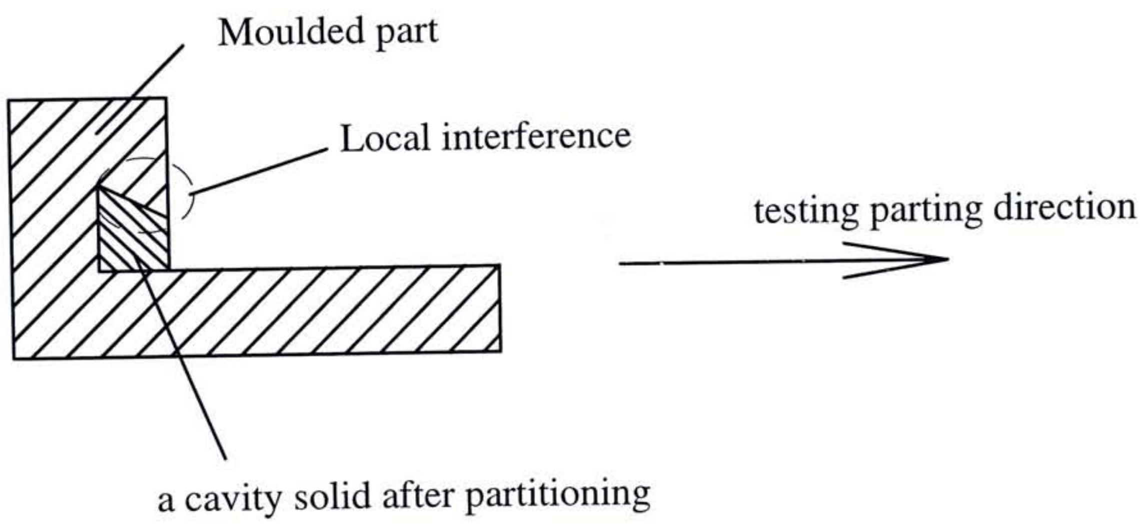


Figure 5.9. An example of local interference.

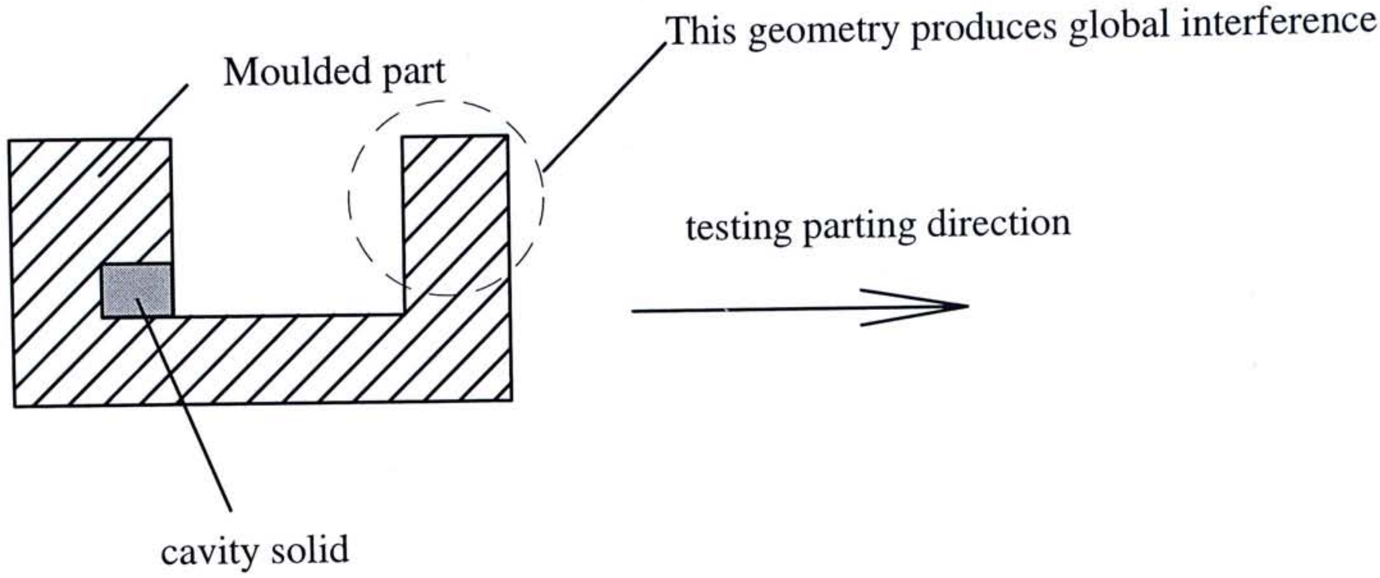


Figure 5.10 . A cavity solid is globally interfered.

The classification is dictated by the existence of lid face and virtual lid face in a cavity solid. A lid face is defined as the face of cavity solid that is located on the convex hull of the moulded part. An example is shown in Figure 5.11.

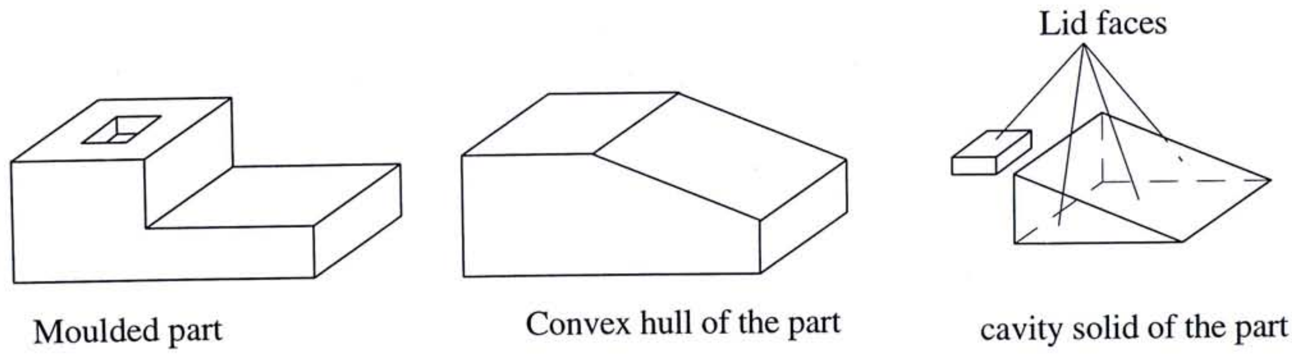


Figure 5.11. An example for showing lid faces

A virtual lid face is defined as the face of the cavity solid that lies neither on the convex hull surface nor the moulded part surface. Virtual lid face is created whenever cleavage of the cavity solid occurs. An example is shown in Figure 5.12.

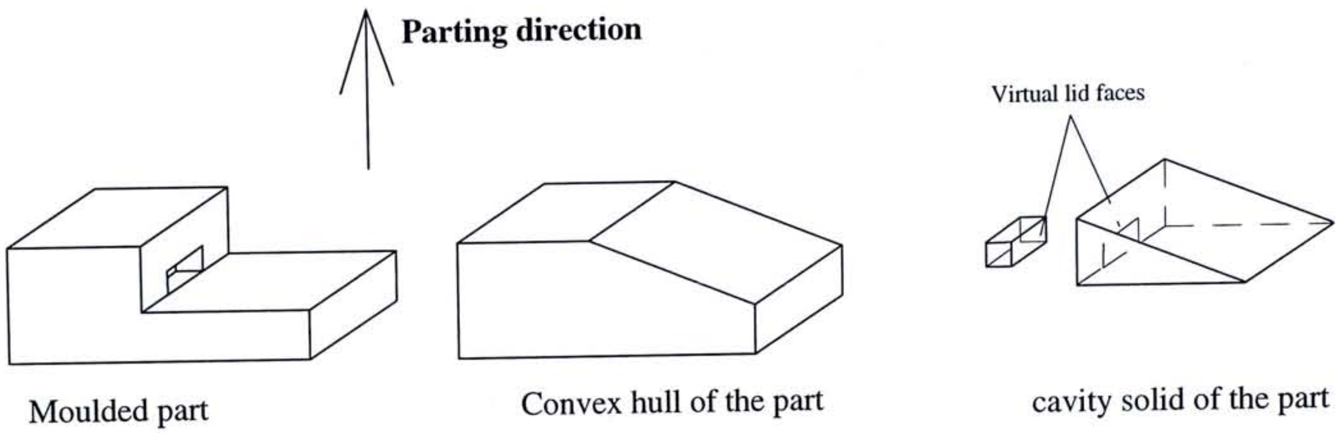


Figure 5.12. An example for showing virtual faces

Generally, cavity solid is classified into the following three main types (Figure 5.13) according to the existence of virtual lid face and lid face:

	Lid face	Virtual lid face
Type 1 cavity solid	✓	✗
Type 2 cavity solid	✓	✓
Type 3 cavity solid	✗	✓

Figure 5.13. Classification of cavity solids

Type 1 cavity solid consists of lid faces but no virtual lid faces. This kind of cavity solid or undercut solid can be resolved if all faces is visible through the lid face. This is because the lid face is a face lying on the convex hull such that global interference never exists. Therefore, Visibility Map is sufficient for blocking test.

Both type 2 and type 3 cavity solid consist of virtual lid faces implying that global interference may exist. In this case, Visibility Map is insufficient for blocking test such that solid sweep may have to be used. However, no global interference test is required if local interference exists for a cavity solid since same result will also be obtained in global interference test. The following table (Figure 5.14) summarizes the above discussion.

	Lid face	**Virtual lid face**	**Visibility check**
Type 1 cavity solid	✔	✘	Vmap only
Type 2 cavity solid	✔	✔	Vmap and then solid sweep
Type 3 cavity solid	✘	✔	Vmap and then solid sweep

Figure 5.14. Summary for visibility check

5.9. Search strategy for parting direction of 2-piece mould

As discussed in previous sections, it is an usual practice to select the main parting direction with the largest projected area while there is no blockage or undercut. In order to speed up the search process, a heuristic search technique based on the preference value as discussed in Section 5.4. is adopted for the search.

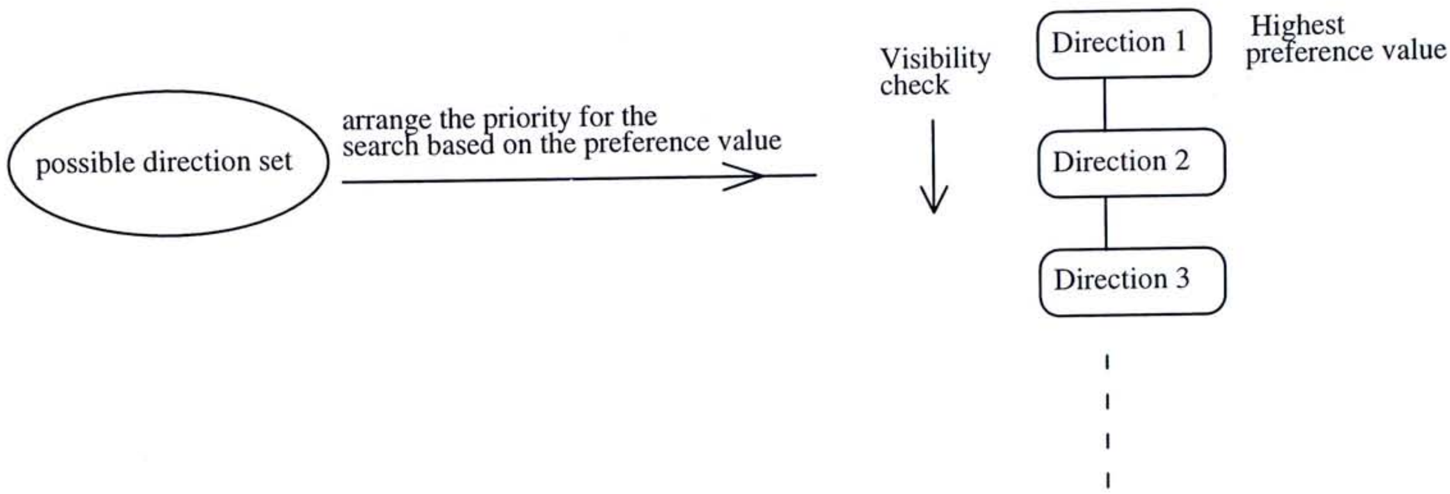


Figure 5.15. Search for main parting direction

The search for main parting direction is depicted in Figure 5.15. and starts with a direction of highest preference value. Visibility check is then applied. If all cavity solids are visible along this direction, then a solution is obtained. Otherwise, the direction with the next highest preference value is tested. The search is terminated when a direction with all cavity solids to be visible is located. If no direction with all undercut resolved can be located, there are two possible cases: the part may not be moulded or, side core or/and split core is required.

CHAPTER 6

Determination of Main Parting Direction and Side Core

In the selection of main parting direction, if there is no proper solution returned, side core has to be considered. In the search for side cores, the objective is to find a main parting direction with minimum number of side cores. While the main parting direction is the one with the maximum possible projected area of the part (Figure 6.1).

The first step in the search for side core is to extract all possible side core directions which are the face normals of the cavity solid. Next, each direction is ranked according to their preference of being chosen as a side core direction. This preference order depends on the degree of blocking (undercut) and the projected area.

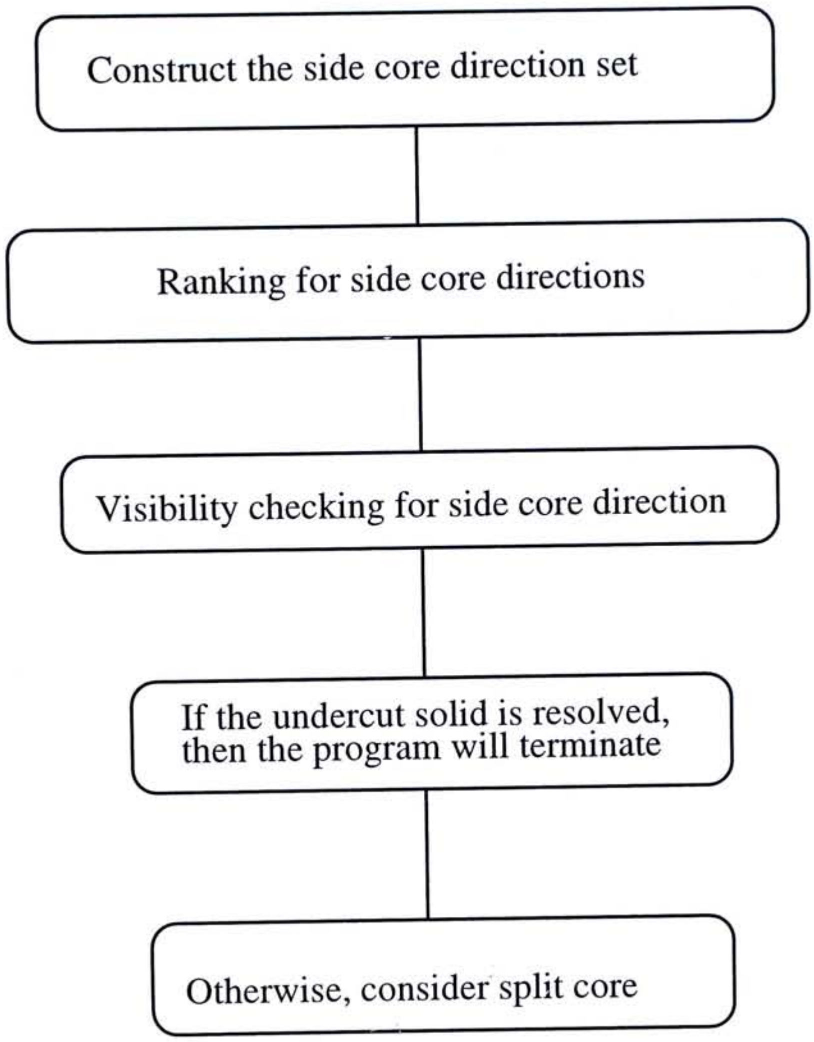


Figure 6.1. The search for side core

6.1. Undercut Evaluation

If a part cannot be moulded by a simple 2-piece mould, the undercut solid associated with a main parting direction is evaluated. A good main parting direction will lead to less degree of blocking and hence, will have a higher preference or possibility to be the final solution. In addition, the degree of blocking is related to the total surface area of the undercut solid. Thus, the surface area of undercuts is selected to be a basis for measuring the amount of undercuts associated with a given parting direction. Larger surface area of the undercut solid will lead to higher degree of blocking.

The evaluation process is similar to those of K.C.Hui and S.T. Tan [ref. 1] except that the total surface area of an undercut is used instead of points on the edges. The process is divided into two steps: Summation of surface area of all undercuts and computation of blocking index. The summation of surface area of all undercut solids excludes the lid faces and the virtual lid faces as they do not involve in blocking.

For the purpose of comparing the amount of undercuts for different parting directions, a *blocking index* is introduced to indicate the amount of undercuts by measurement of the relative amount of obstruction for a possible direction. The method for computing blocking index is shown below :

$$\text{Blocking index} = \frac{\text{Summation of surface area of all blocked cavity solids except the lid faces and the virtual lid faces}}{\text{Total surface area of cavity solids except the lid faes and the virtual lid faces}}$$

where blocking index $\in [0,1]$

According to the above discussion, Blocking index and Preference value are integrated to determine the sequence of main parting direction for subsequent test.

6.2. Determination of main parting direction

An discussed in previous sections, the number of side cores in a mould should be kept to a minimum in order to reduce the mould production and maintenance costs. Thus, the main parting direction would be preferably the one that leads to maximum preference value and minimum blocking index. Unfortunately, this solution may not exist in many cases. To compromise these two critical factors for the selection of main parting direction, a parameter, Main Parting Index (MPI), is used which is formulated as below:

Main Parting Index (MPI) = Preference Value * (1 - Blocking Index)

A large projected area (preference value) will thus lead to a large MPI value while a low degree of blocking (Blocking Index) will also lead to a large MPI value. According to the MPI values of a main parting direction, a sequence of priority for the selection of main parting direction can be constructed.

6.3. Determination of side cores for a given main parting direction

The side core direction set is obtained from two sources which is different from the extraction of main parting direction set. One is the face normals of the cavity solid. This is because the direction of the side core is dependent on the geometry of the cavity solid only. Moreover, for oblique undercut (Figure 6.2), consideration of side core directions parallel to face normals of undercut solid may not be sufficient for resolving the undercut.

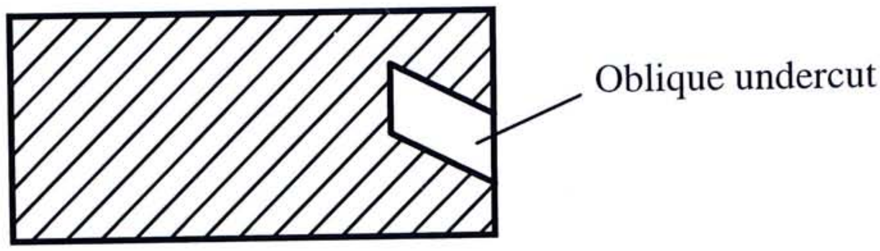


Figure 6.2. Example of oblique undercut

In this case, additional source of directions are added to the testing direction set. These directions in the same direction of the undercut solid's edges which consist of a vertex on the edges of lid faces (Figure 6.3). Obviously, this set of direction enables the side core to have a motion along the edge directions. These directions are merged into the direction sets and the duplicated ones are eliminated.

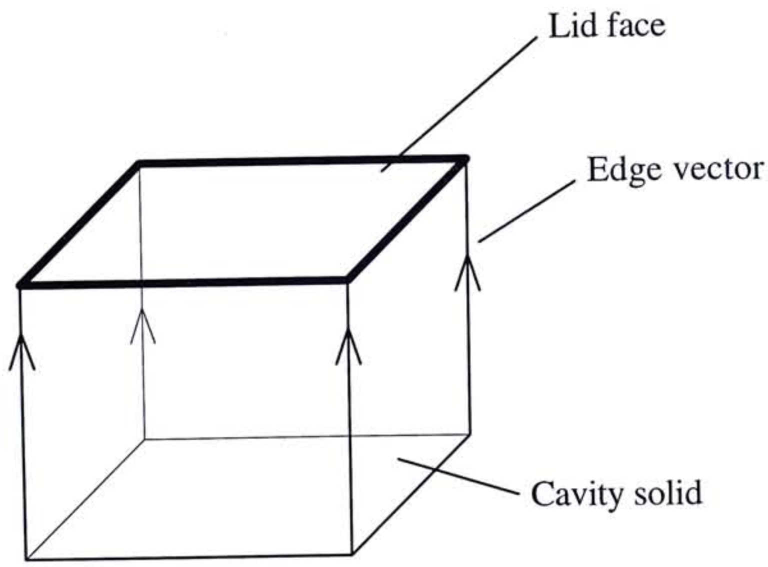


Figure 6.3. Extraction of edge vectors

In general, side core directions are preferably orthogonal to the main parting direction such that the mechanism for guiding motion of side cores can be easily implemented. To achieve this, the side-core preference value (SPF) [ref. 2] is adopted which is the dot product of the unit vectors of the main parting direction and the side-core direction as stated below:

SPF = Dm·Ds

where SPF = side-core preference value.

Dm = unit vector of main parting direction.

Ds = unit vector of side core direction.

This value is used to measure the degree of orthogonality of the side core direction with the main parting direction. The lower the SPF values of the side core direction, the higher is the preference of this direction for being chosen as the solution. Thus, based on the SPF values, a priority sequence of possible side core directions can be produced. The test for side core directions can then be performed according to this priority sequence.

A local and a global interference tests are then performed on the cavity solid. If interference occurs, the testing direction is discarded since it implies that there is blocking in the trajectory of the side core motion. Otherwise, this testing direction is saved. The above procedure is repeated for the next possible side core direction until all cavity solids are cleared out. If no solution exists, another set of side core directions from the next main parting direction will then be tested until a solution is found or all main parting directions are exhausted.

6.4. Search strategy for main parting direction and side core direction

6.4.1. The search for single side core

The search process for single side core is similar to those for the search of main parting direction in a 2-piece mould except that the MPI value is used as the search basis. A breadth-first search strategy is adopted for the search.

Before starting the search, the possible main parting directions are arranged in descending order of their MPI values. A search tree is then constructed. A number of nodes representing side core directions obtained from the face normals of the cavity solids are constructed for each main parting direction (Figure 6.4). These nodes are constructed such that their directions are not parallel to that of their parent nodes. Similar to the selection of main parting direction, the search priority of the side core directions are ranked based on the SPF value.

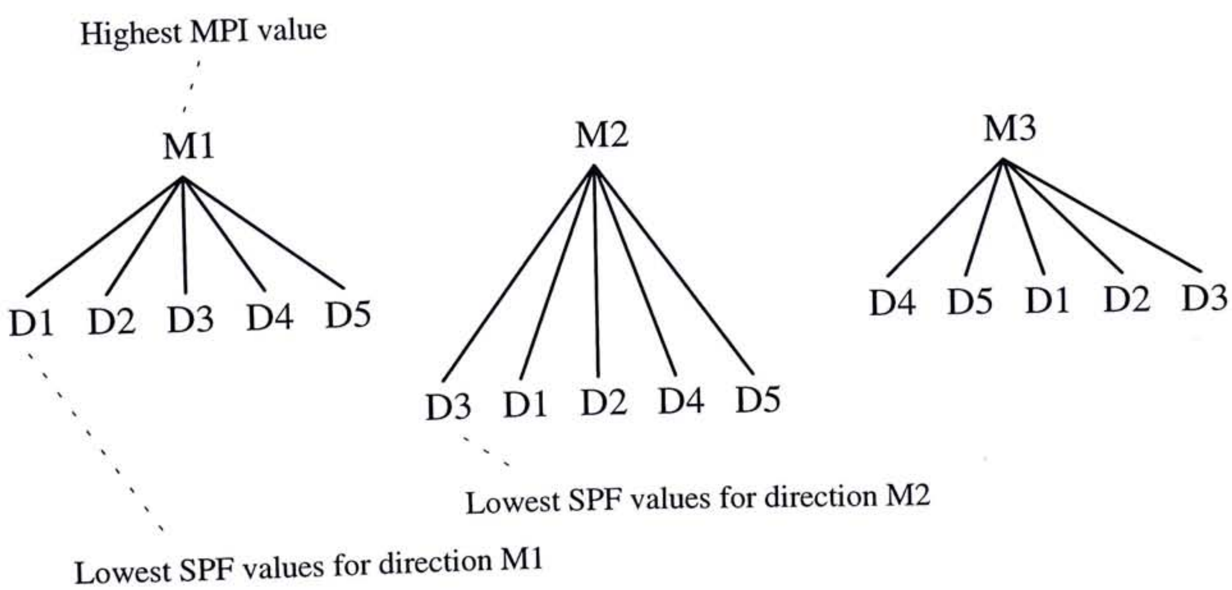


Figure 6.4. Example of a search tree

The main parting direction with highest MPI value (M1 in Figure 6.4) is then examined. Its leaf nodes are considered one by one according to the pre-defined sequence. Note that visibility check for the leaf nodes is based on the cavity solid remained for the selected main parting direction. Thus, each root will have its own cavity solid. Hence, the same set of cavity solid will be used for the leaf nodes of the same root. Three possible results are identified as listed below.

Type 1 result. All cavity solids are resolved
If a side core is found with all cavity solids being resolved, then the search will terminate. This side core and its corresponding main parting direction will become the required solution.

Type 2 result. No improvement in resolving cavity solid
If a side core direction does not lead to any improvement in the reduction of cavity solid, then the direction is invalid and must be discarded so that it will not be considered in subsequent search.

Type 3 result. Cavity solid is partially resolved
If a side core direction leads to a reduction of the cavity solid, then this direction and its remaining undercut solid are saved for further processing.

Classification of the results in the search for side core is to be used for creating further branches of the search tree if multiple side core is to be used. If Type 1 result does not exist after all leaf nodes of the first root (M1 in Figure 6.4) are visited, then similar search is applied to the leaf nodes of the next root (M2 in Figure 6.4). If all leaf nodes are visited while no solution is returned, multiple side cores may have to be considered.

6.4.2. The Search for Multiple Side Cores

Locating multiple side cores by examining all possible combination of different side cores is a straightforward but time-consuming approach. In order to speed up the search, the classification result in the preceding search level plays a decisive role in the construction of the search tree. Only nodes with Type 3 result will produce a number of son nodes for further search. All nodes in further branches are thus the sons of those nodes with Type 3 result in previous search level. An example is shown in Figure 6.5.

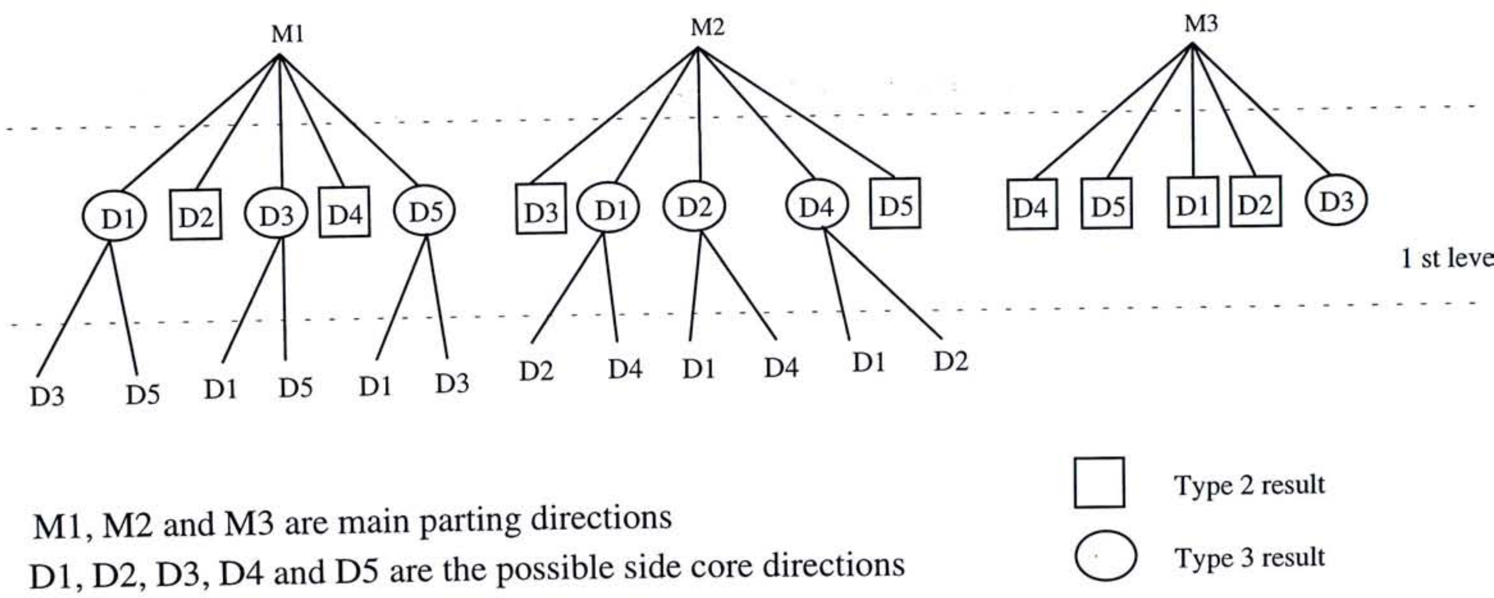


Figure 6.5. Generation of search tree

Consider M1 in Figure 6.5, the side cores D1, D3 and D5 are all assumed to give out the Type 3 result and are marked with circles. Branches are thus created from D1, D3 and D5. The sons of D1 are D3 and D5. Similarly, the sons of D3 are D1 and D5. In addition, the sons are not randomly positioned. They are arranged according to the sequence in the first search level which is D1, D3 and D5 for M1. This ensures that the sons preserve the properties of the search priority.

Besides, when there is only one node with Type 3 result (M3 in Figure 6.5), no branch is generated and in turn, no further side core search is performed. However, D3 will be saved for further usage in the search for split cores.

After determining the global plan of second level of the search tree, the search will start. If a solution is still not found, then the leaf nodes will generate further son nodes for next search level according to the result of the nodes.

Finally, after examining all possible search levels, if no solution is returned for resolving all cavity solid, split core may have to be considered.

CHAPTER 7

Determination of Split Core Direction

In previous sections, parts with external undercuts are considered. However, internal undercut is a common feature in an injection moulded component. Thus, it is essential to develop algorithm for handling internal undercuts.

To mould a part with internal undercut, split core provides an effective method. As stated in Section 2.3.2., split core is a mechanical device which moves away from the internal undercut when the mould opens. This mechanical device increases the tooling and maintenance costs so it is undesirable and should be avoided as far as possible.

7.1. Overview of determination of split core direction

Split core is to be considered when a part cannot be moulded by the use of side core only. The algorithm for locating split core directions is similar to that for side cores in which the set of test directions is composed of directions from two sources. One source is the face normals of the cavity solid, and the other is the edge vectors of the cavity solid. Since the criterion for selecting split core direction is similar to that for side core selection, the SPF value is used to rank the directions for subsequent tests. The algorithm analyses the remaining undercut solid by the methods of solid sweep and Visibility Map, and finally, the number of split core and its direction are returned. If a valid solution is not obtained by considering the use of split cores, then the part cannot be moulded.

7.2. Visibility check for split core

As discussed in Chapter 4, the methods of visibility check is based on solid sweep and Visibility Map. Solid sweep plays a major role in the test for split core direction, and is slightly modified to suit the purpose.

The sweep distance for the interference test of side core is a very large arbitrary pre-set value whereas the sweep distance for interference test of split core depends on the geometry of the undercut which will be discussed in the following sections.

The process for interference test is divided into two steps. The first is to test for local interference of the undercut solids along the test direction. It is performed with a Visibility Map. The next step is to detect global interference in which solid sweep is used. The objective of the Visibility Map is to serve as a preliminary test to detect local interference of the undercut solid so that no time-consuming solid sweep is applied if local interference exists. If there is no local and global interferences for an undercut along a direction, this direction will be saved for further tests.

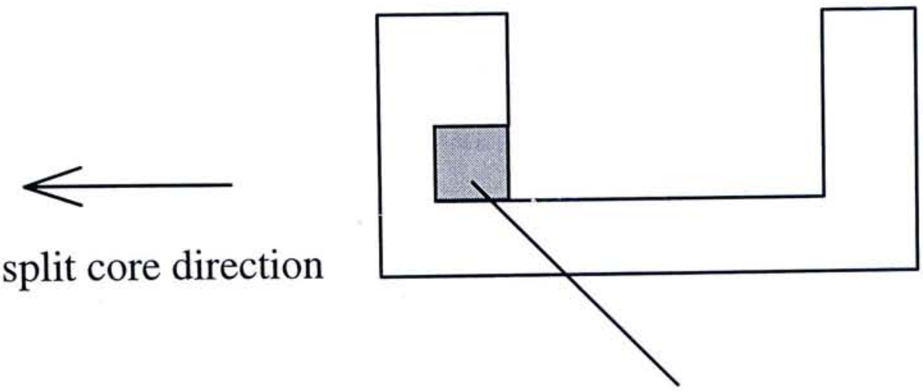


Figure 7.1. Invalid direction for the undercut solid

7.3. Selection criteria for split core

As discussed in Section 2.3.2, the split core motion is a two-stepped process (Figure 7.2). The first step is to remove the split core from the undercut feature. The second step is to move the split core away from the part. When the split core completes its motion without interference, the split core is regarded as valid.

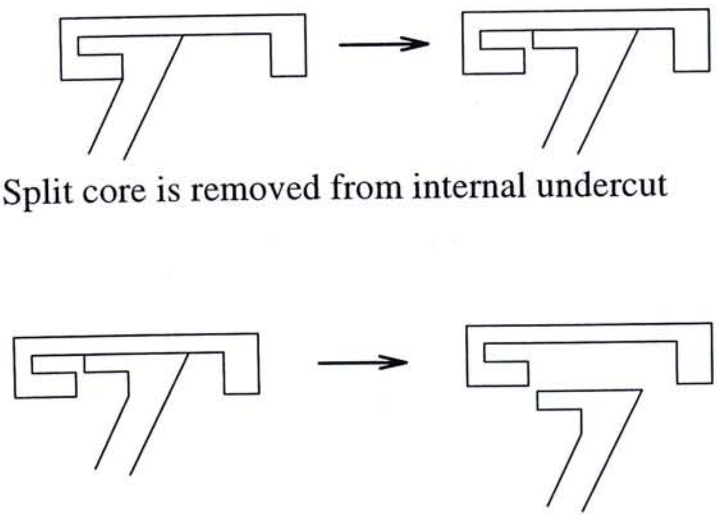
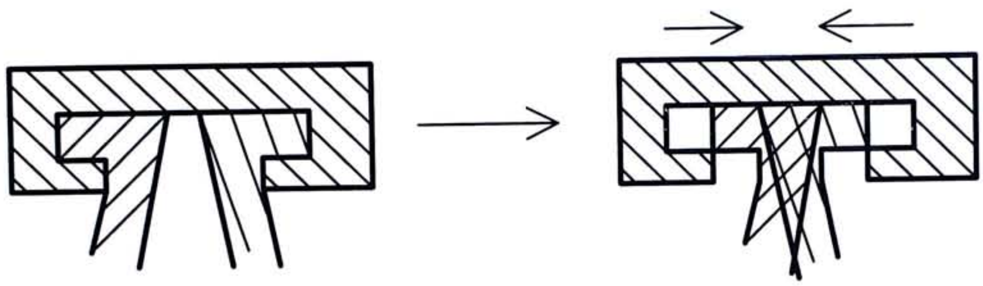


Figure 7.2. Principle of split core for detecting internal undercut.

This is only true for the case of single split core. There may be cases when the number of the valid split cores is more than one such that their motions may interfere with each other (Figure 7.3). Hence, if multiple split cores are used in a mould, further interference test have to be performed.



Interference exists when the split cores move

Figure 7.3. Interference between split cores

7.4. Trajectory of split core

The trajectory of a split core is to be tested against a part for any global interference along those directions that passed the preliminary local interference test. Since the trajectory of a split core is composed of two paths, one is the path for the split core to move away from the undercut, and the other is the path for split core to be removed completely from the part, two types of solid sweep, *primary* and *secondary solid sweep*, are introduced to deal with the two steps of split core motion.

7.4.1. Primary Solid Sweep

A primary sweep is performed for testing the first path of trajectory of a split core when the split core is being removed from an internal undercut. In this case, the distance of sweep depends on the depth of undercut and the thickness of the split core handle, and is expressed as

$$Dw = Du + Dh$$

where Dw : sweep distance

Du : depth of undercut

Dh : thickness of split core handle.

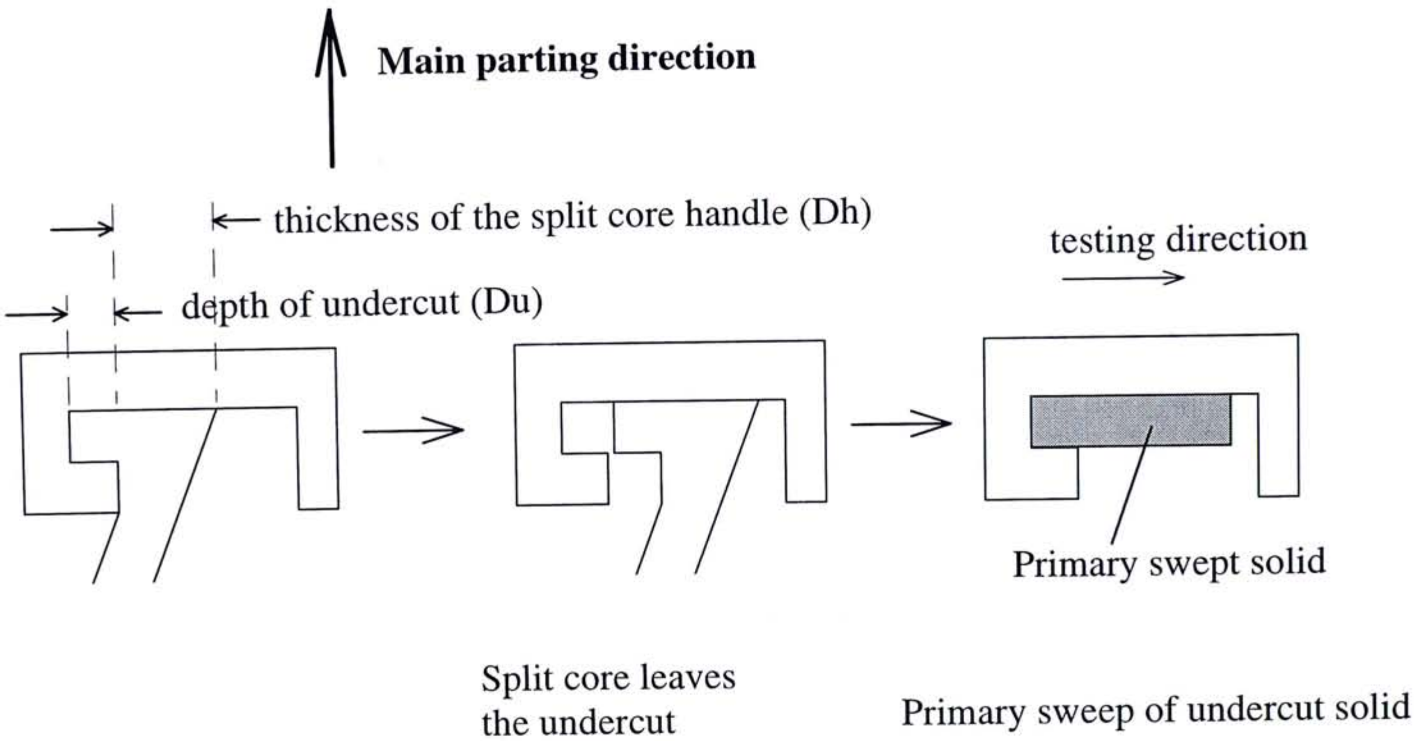


Figure 7.4. Primary solid sweep

The depth of undercut is determined by calculating the maximum length of the undercut solid along the testing split core direction. Whereas, the split core handle is a part attached to the undercut (Figure 7.4). The thickness of the split core handle is user-defined. The minimum thickness for general mould design is assumed. The swept solid is obtained by taking the union of the regions created by sweeping of each face of the undercut solid along the candidate split core direction. If the swept solid interferes with the moulded part (Figure 7.5), then this direction is invalid for resolving this undercut.

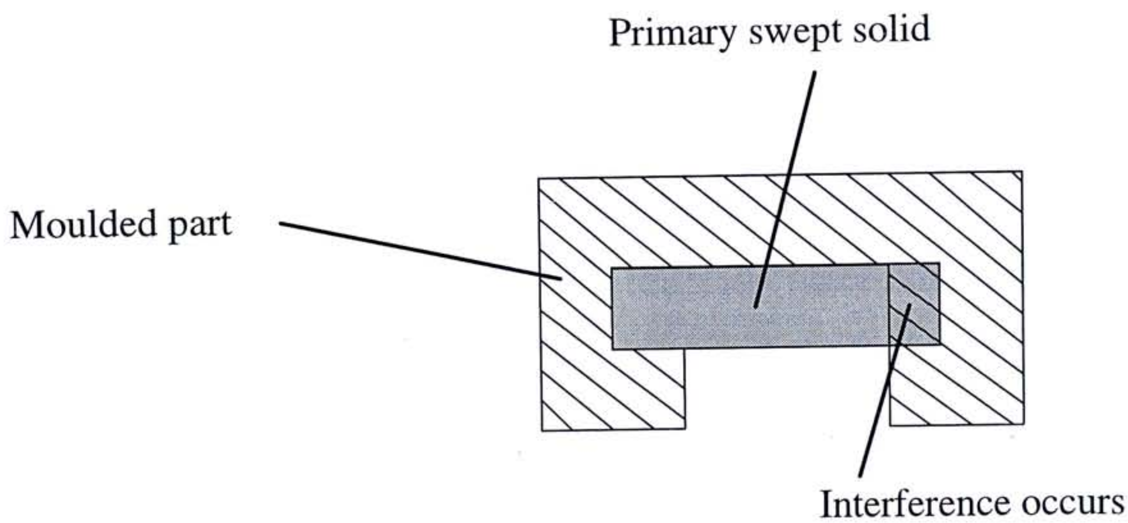


Figure 7.5. Interference occurs in primary solid sweep

However, if the resultant swept solid is free of interference, the direction may not be a valid solution since the split core may be obstructed to leave the part along the main parting direction. A secondary solid sweep is thus used to test obstructions along the main parting direction which will be discussed in the following section.

7.4.2. Secondary Solid Sweep

The secondary sweep process is applied if a direction is preliminarily proved to be feasible by the primary solid sweep. Before applying the secondary solid sweep, the undercut solid is subtracted from the primary swept solid in order to resemble the split core at the final position of the primary sweep. A secondary sweep is then performed on the solid obtained. This is attained by applying a sweep operation on each face of the solid along the main parting direction. The sweep distance is a constant that is large enough to extend beyond the moulded part. Interference check between the moulded part and the secondary sweep solid is then performed. Since the main parting direction is composed of two opposite and parallel directions, two possible sweep directions are required to be considered. If one of the directions fails, the other one has to be tested. If both directions fail, the split core in consideration is invalid. The Figure 7.6 illustrates an example of secondary solid sweep.

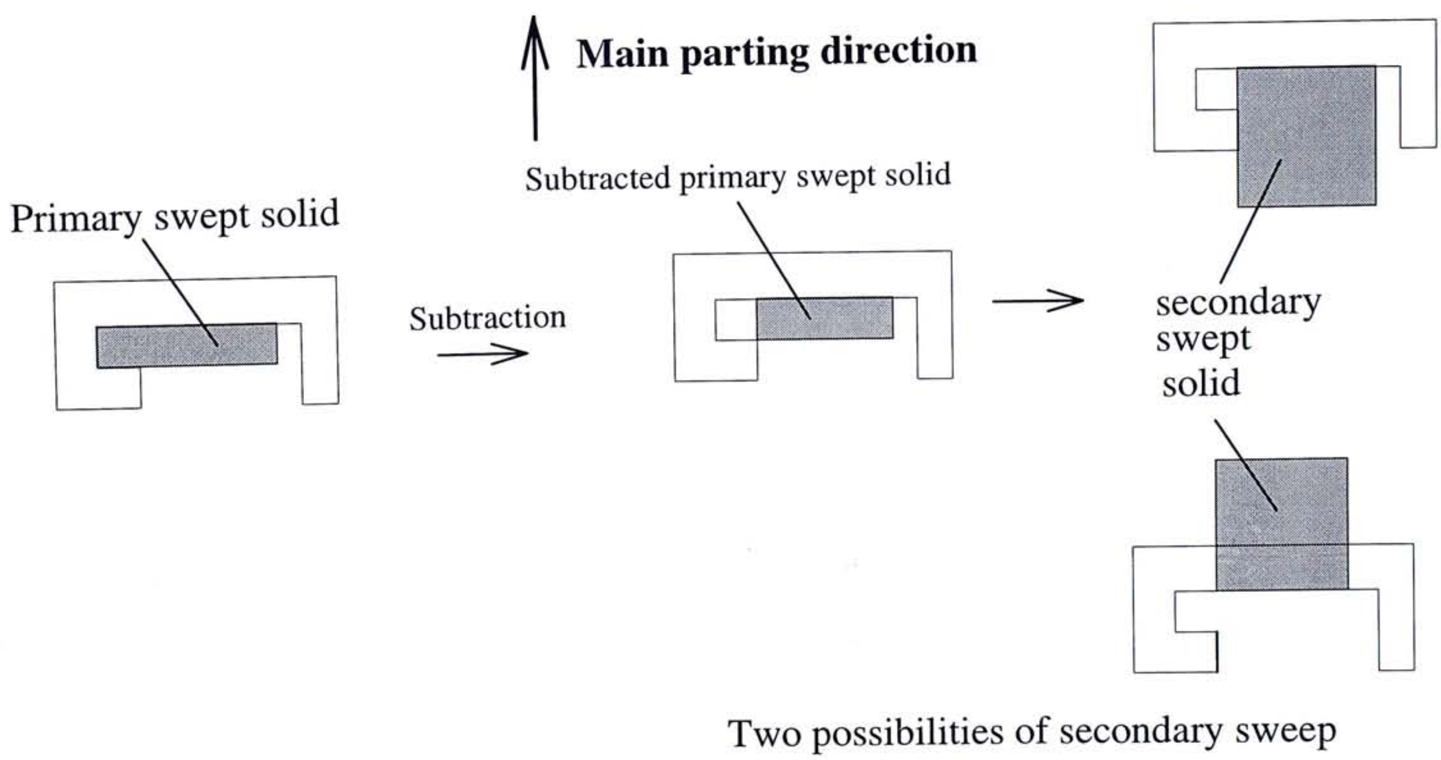


Figure 7.6. The secondary solid sweep process

7.5. Interference between split cores

The above mentioned method is sufficient for handling parts that require a single split core. For parts that require multiple split cores, interference between split cores may exist when all split cores are in motion simultaneously during opening of the mould. Test is required to detect interference among split cores. Secondary swept solids constructed with different directions are considered for interference test.

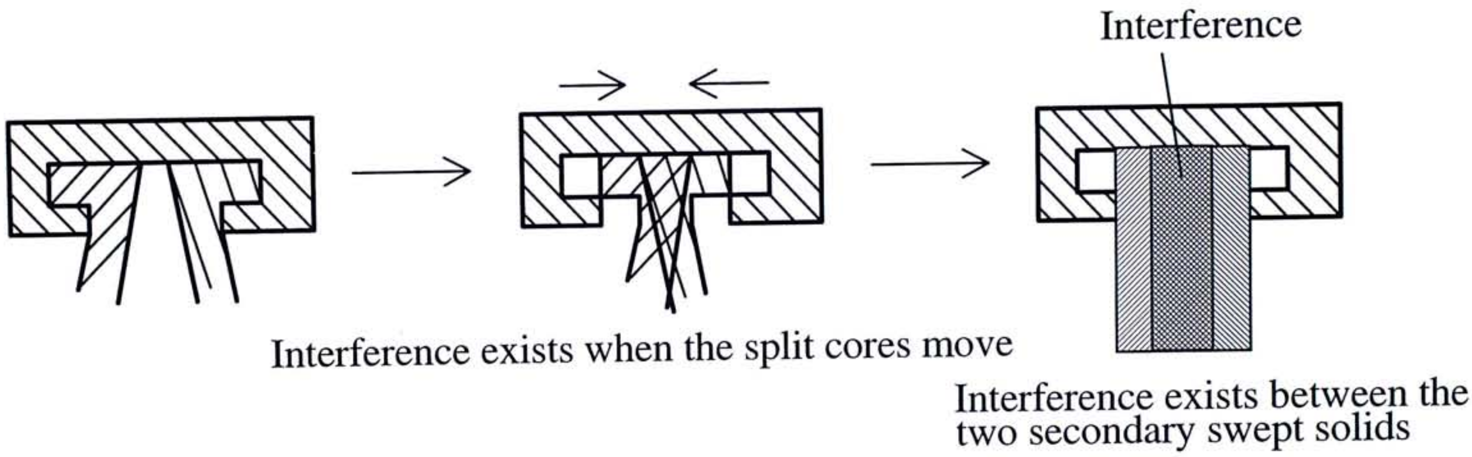


Figure 7.7. Detecting the interference between split cores

7.6. Search Strategy for Split Core

Similar to the selection of side cores, the problem of selecting split core is to locate a combination of main parting, side core and split core directions. The least number of side cores and split cores is preferred.

7.6. Search Strategy for Split Core

Similar to the selection of side cores, the problem of selecting split core is to locate a combination of main parting, side core and split core directions. The least number of side cores and split cores is preferred.

In the selection of side cores, the set of undercut solids for different combinations of main parting direction and side cores are obtained. These undercut solids are also used for the selection of split cores. The possible split core directions are arranged in ascending order of SPF value as stated in Section 6.3. The search is started by considering only the cases with the least number of side cores in each main parting direction. By counting the number of nodes from the root in a main parting direction to a leaf node, a combination of directions with the least number of side cores can be obtained. Hence, a leaf node is also referred to as a node with all its son nodes returning Type 2 results (no part of the cavity solid is removed). The undercut solid of the leaf nodes are then used for the search of split core. There is an important property that the cavity solids in all leaf nodes of a main parting direction are the same. Therefore, the split core test will apply to only the case with the least number of side cores in each main parting direction. Besides, since split core must be cleared out of the moulded part along the main parting direction, different main parting direction will give different solution. Suitable ranking strategy for the priority of the main parting directions is required which is based on the MPI value of the main parting direction and the least number of side core in the main parting direction. The search strategy for split core is similar to that for a side core except that interference between split cores has to be considered. Whenever a combination of split cores is obtained such that the cavity solid is cleared, an interference test between split core is performed. If interference exists, the search will continue. Otherwise, a solution is obtained.

An example of a search tree for side cores is shown in Figure 7.8. In this example, the leaf nodes {D5 of M1}, {D1 of M2} and {D3 of M3} will be considered since they are the combinations with the least number of side cores in their corresponding main parting directions. There is no offspring for {D3 of M3} since all directions except D3 are invalid. By comparing the number of side cores among the three cases, the case with least number of side cores will have the highest priority for split core test. However, if they all have the same number of side cores, the case having the highest MPI value {D5 of M1} of its main parting direction will be tested first. The addition of split cores are tested for these combinations of directions in the order as listed. If all undercuts can be resolved by one of the above cases, then interference test for the possible combination of the split core direction is performed. If there is interference, the search will continue. Otherwise, a solution is found. Finally, when no solution is found while all possible cases are examined, it can be concluded that the part cannot be moulded.

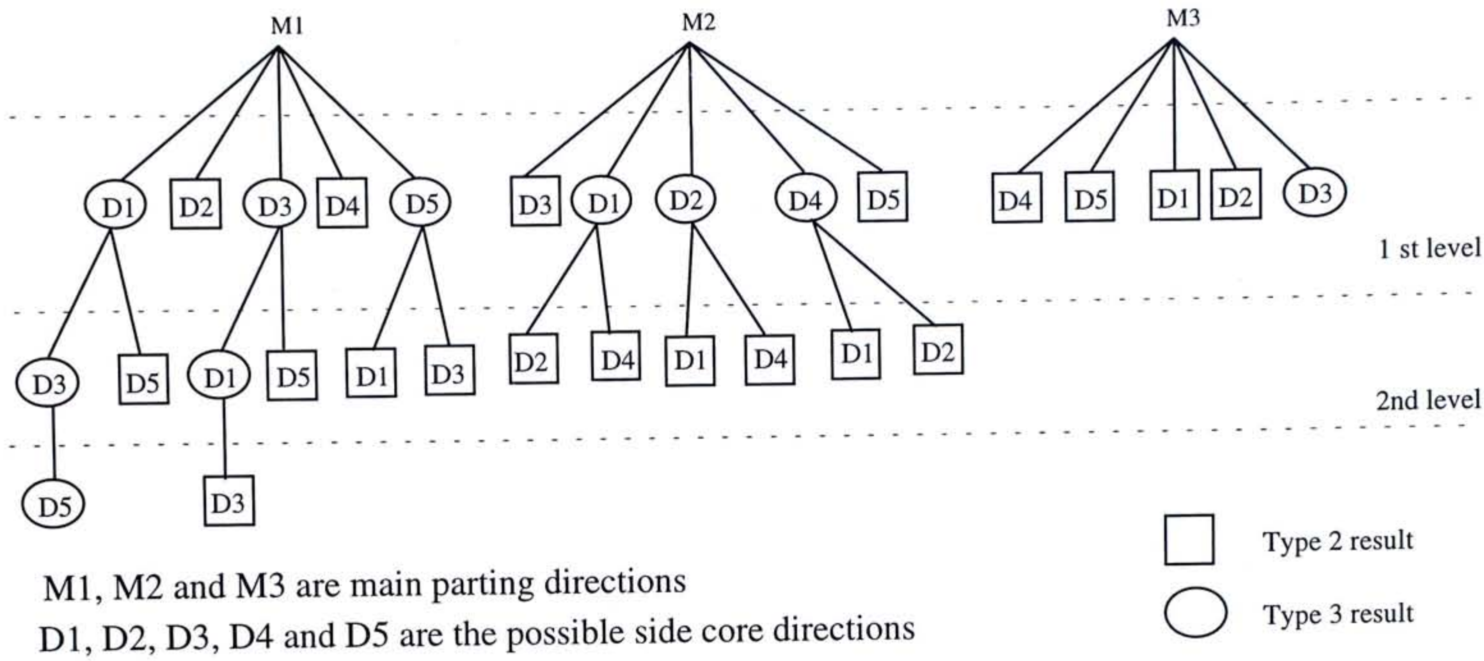


Figure 7.8. An example of search tree for side core

This search strategy ensures the final result to be a feasible one with minimum number of split core and side core as the search is started from the case of minimum number of split core.

CHAPTER 8

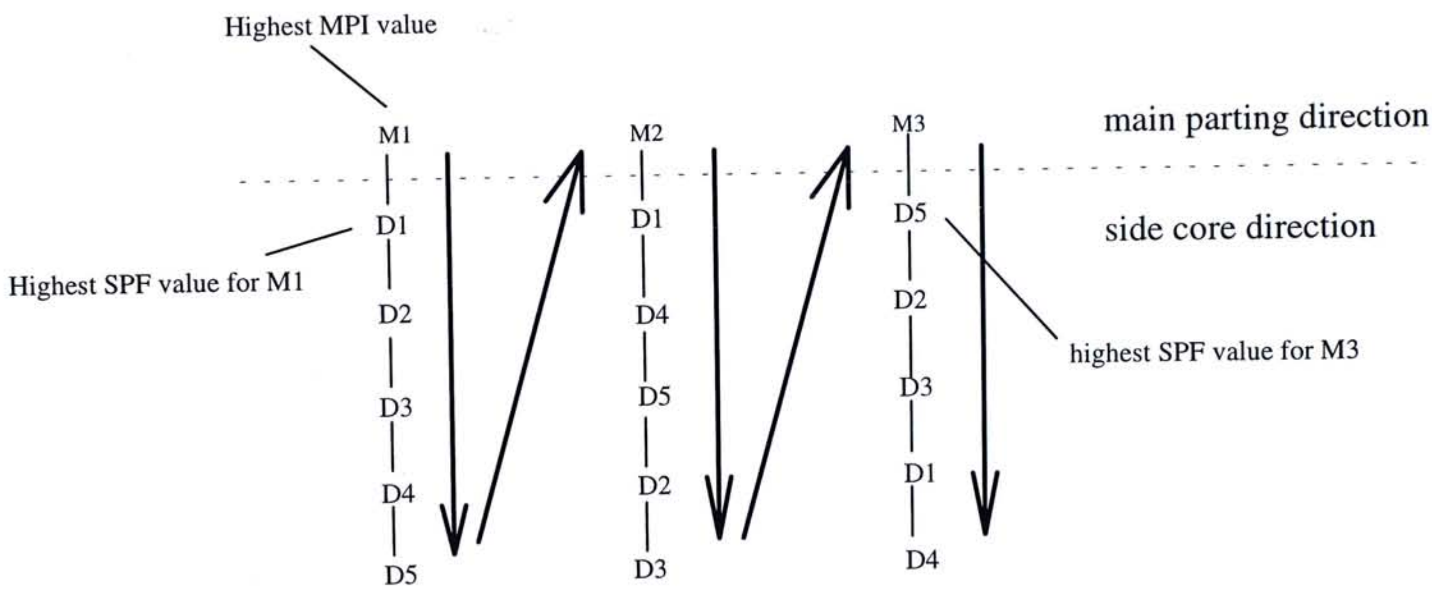
A Heuristic\Depth-First Search Strategy

A breadth-first search is known to be more time-consuming than a depth-first search. In order to compare the performance of the two approaches, a depth-first search strategy for selecting a combination of main parting, side core and split core direction is also implemented.

8.1. Side core determination

Depth-first search is a well-known fast search method. In order to obtain a more reliable result in shorter time, heuristics are added. Generally, the heuristic is the same as that of the breadth-first search as discussed in previous chapters. The main parting directions are arranged in descending order of the MPI value and the side core directions are arranged in descending order of the SPF value.

Initially, the direction with the highest MPI is selected. Side cores are then added according to their SPF values and the cavity solids are evaluated. If not all cavity solids are resolved (Type 1 result), then the direction with the next highest MPI is selected. The whole procedure is repeated until a Type 1 result is obtained or all combinations have been tested. Figure 8.1 shows an example of the depth-first search method in which the arrows show the search sequence. Similar to the breadth-first search, the direction with Type 2 result (i.e. no reduction of cavity solid) are saved (for split core combination). While directions with Type 3 result will be discarded since it has no contribution in resolving undercut.



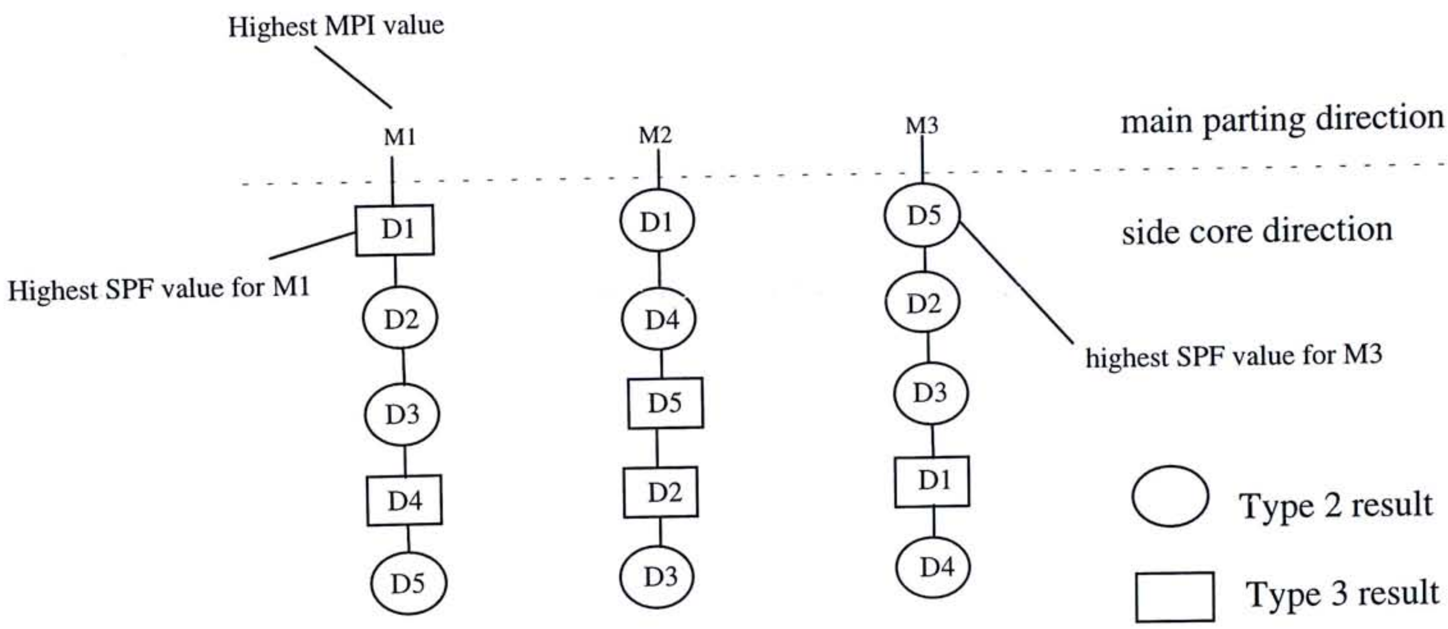
M1, M2 and M3 are main parting directions
D1, D2, D3, D4 and D5 are the possible side core directions

Figure 8.1. Example of depth-first search strategy

Finally, if no solution is found for resolving all cavity solid or no Type 1 result is found, each main parting directions (M1, M2 and M3) will collect their own valid side core direction sets which will be used for split core combination.

8.2. Split core determination

In order to fulfill the requirement of the least number of side cores, valid side core direction sets of different main parting directions are compared. The one with the least number of side cores will have the highest priority to be considered for the addition of split core.



M1, M2 and M3 are main parting directions
D1, D2, D3, D4 and D5 are the possible side core directions

Figure 8.2. Example of heuristic\depth-first search strategy

An example is shown in Figure 8.2. In the example, there are three side core directions of M1 that give Type 2 result. There are also three and four side core directions of M1 and M3 respective that give Type 2 result. Although both M1 and M2 consist of the same number of side cores, M1 is preferred since the MPI value of M1 is higher.

Split core directions are then considered. The testing sequence is defined in a similar way to that of the side cores. The undercut solids used in split core tests are those remained in the side core consideration. If Type 1 result (i.e. all cavity solids are resolved) is obtained, a solution is found. Otherwise, the part is regarded as not mouldable.

CHAPTER 9

Experimental Results

Introduction

In this project, two experimental systems based on breadth-first search and depth-first search strategies are developed and tested. Both of them are implemented with C language and are integrated to a geometric kernel CV-DORS (rel. 2.0) running on a SUN SPARC 10 Model 30 workstation. The test models are constructed with an existing CAD package CADDS5 and the result is also visualized through CADDS5.

The geometric kernel CV-DORS (ComputerVision Developers Open Resource Software) provides a set of interfaces that enable user to create, modify and analyse geometric models. It also gives user a programming access to CADDS5 data base. The package provides a board range of different routines for many applications such as NURBS curves and surfaces creation, boolean operations for solid modeling, vector manipulation and sweep operations. In addition, the package is also interfaced with C, C++ or Fortran. User can invoke the routines in a convenient way to construct a complex geometric model. Thus, CV-DORS are selected for implementation in this research.

In the following, several test pieces are tested by the breadth-first search and the depth-first search approaches. The screen displays of the results are shown and the overall CPU times are recorded. The times for individual processes are also tabulated for comparison.

Sample run

Case 1 (Rectangular block with internal and external undercuts)

A rectangular block with an internal undercut and two external undercuts as shown in Figure 9.1 is tested by the experimental systems. The final results of both methods are identical and is shown in Figure 9.1. The symbols E, M and I denotes side core direction, main parting direction and split core direction respectively.

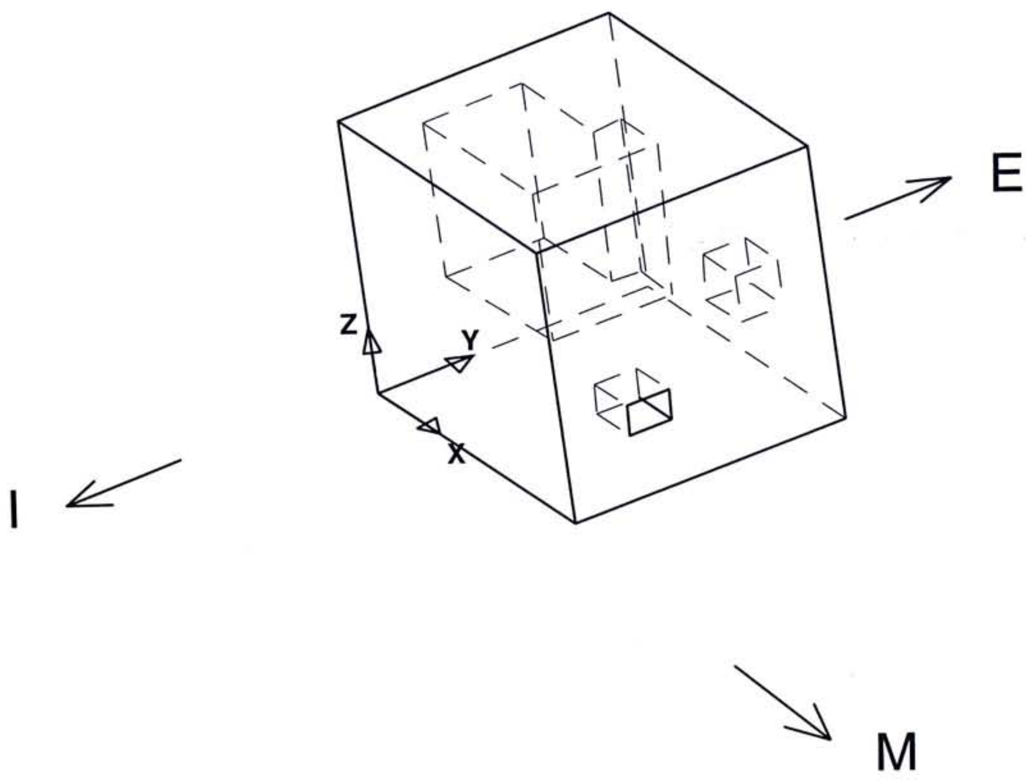


Figure 9.1. The final display result from both methods for case 1

The cavity solid of the part is composed of three separate solids as illustrated in Figure 9.2.

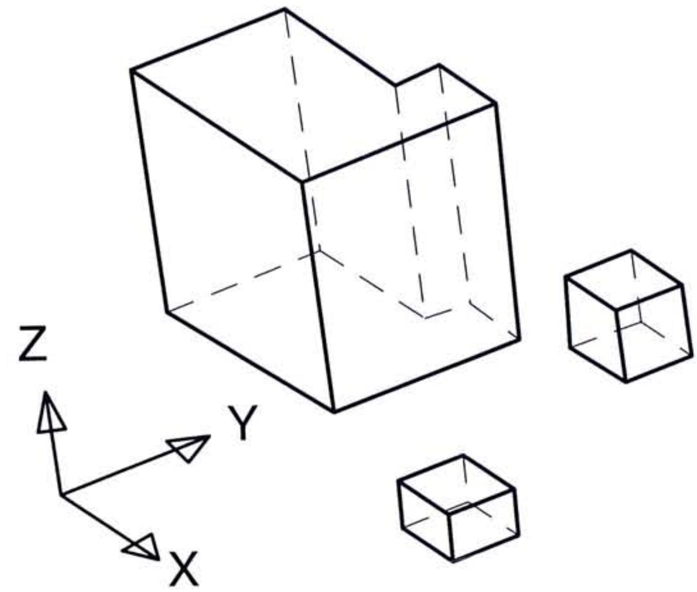


Figure 9.2. The cavity solids of case 1

On exiting the module for determining the main parting direction for simple 2-piece mould, the preference value, blocking index, and MPI values of the main parting directions are recorded and are listed in Table 1. In addition, the processing times for some individual stages are recorded and listed in Table 2.

	Main parting direction	**Preference value**	**Blocking index**	**MPI value**
1st priority	(1,0,0)	1	0.228	0.772
2nd priority	(0,1,0)	1	0.936	0.064
3rd priority	(0,0,1)	1	1	0

Table 1. The priority sequence of main parting directions for the mould having side core(s) (case 1).

	Breadth-first search method	Depth-first search method
Solid copy	124 sec.	43 sec.
Cut solid	81 sec.	36 sec.
Boolean operations	68 sec.	29 sec.
<u>Processing time spent in each stage</u>		
Cavity solid creation	15 sec.	15 sec.
Determination of main parting direction for 2-piece mould	148 sec.	148 sec.
MPI value computation	96 sec.	96 sec.
Re-union of remaining cavity solids	16 sec.	16 sec.
Search for side cores	682 sec.	267 sec.
Search for split cores	48 sec.	48 sec.
Total CPU time	1164 sec.	644 sec.

Table 2. The processing times for individual stages

Case 2 (chassis of a light switch)

The light switch is a rectangular shell with a through hole on the top face and two blind holes on its inner side wall. The test results with both methods are identical and are shown in Figure 9.3. Two split cores are required as shown by the arrow I and the main parting direction is a vector along the Y-axis as shown by the arrow M. Cavity solid(s) of the part are also shown in Figure 9.4.

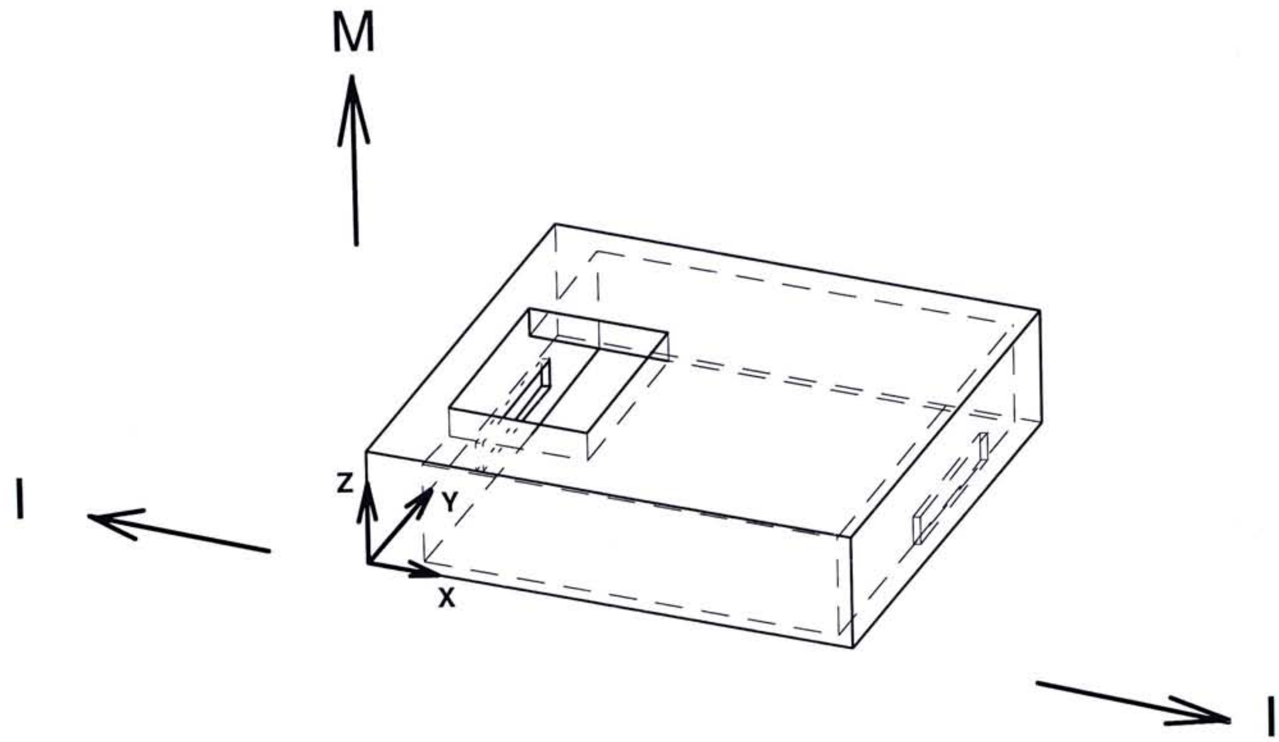


Figure 9.3. The final result of both methods for case 2

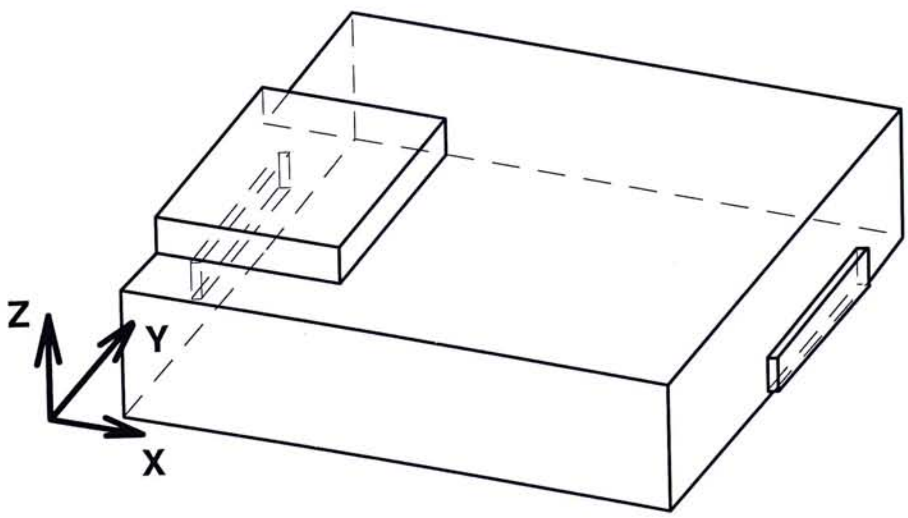


Figure 9.4. The cavity solid of the part in case 2

	Main parting direction	Preference value	Blocking index	MPI value
1st priority	(0,0,1)	1	0.0411	0.9589
2nd priority	(0,1,0)	0.625	0.9589	0.0257
3rd priority	(1,0,0)	0.5	0.9569	0.0215

	Breadth-first search method	Depth-first search method
Solid copy	156 sec.	119 sec.
Cut solid	91 sec.	52 sec.
Boolean operations	135 sec.	117 sec.
<u>Processing time spent in each stage</u>		
Cavity solid creation	15 sec.	15 sec.
Determination of main parting direction for 2-piece mould	703 sec.	703 sec.
MPI value computation	131 sec.	140 sec.
Re-union of remaining cavity solids	86 sec.	85 sec.
Search for side cores	1486 sec.	792 sec.
Search for split cores	70 sec.	70 sec.
Total CPU time	2614 sec.	1914 sec.

Case 3 (Solid with recesses)

A test piece with a rectangular recess and a T-shaped recess is shown in Figure 9.5.

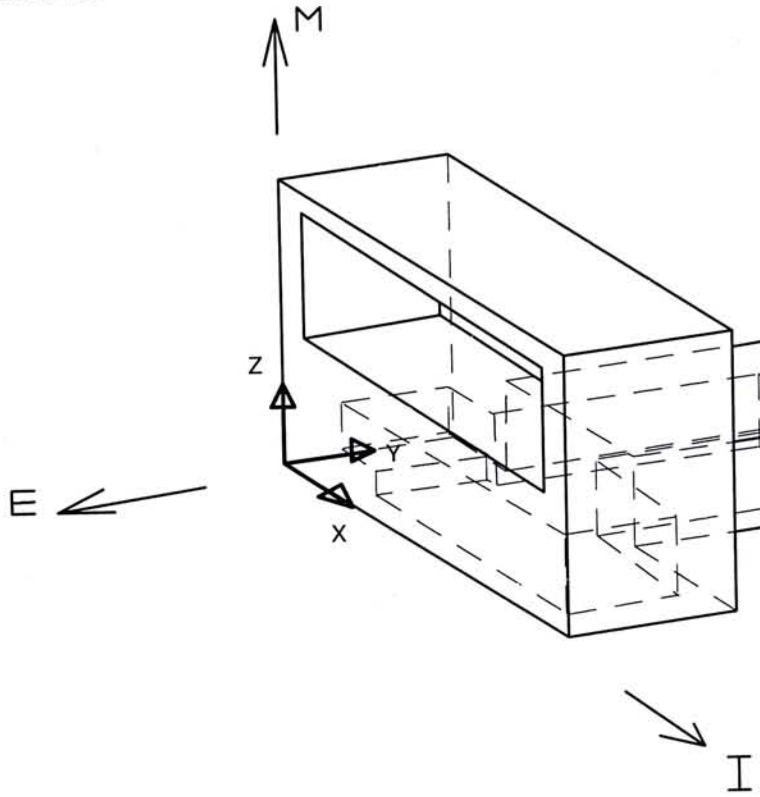
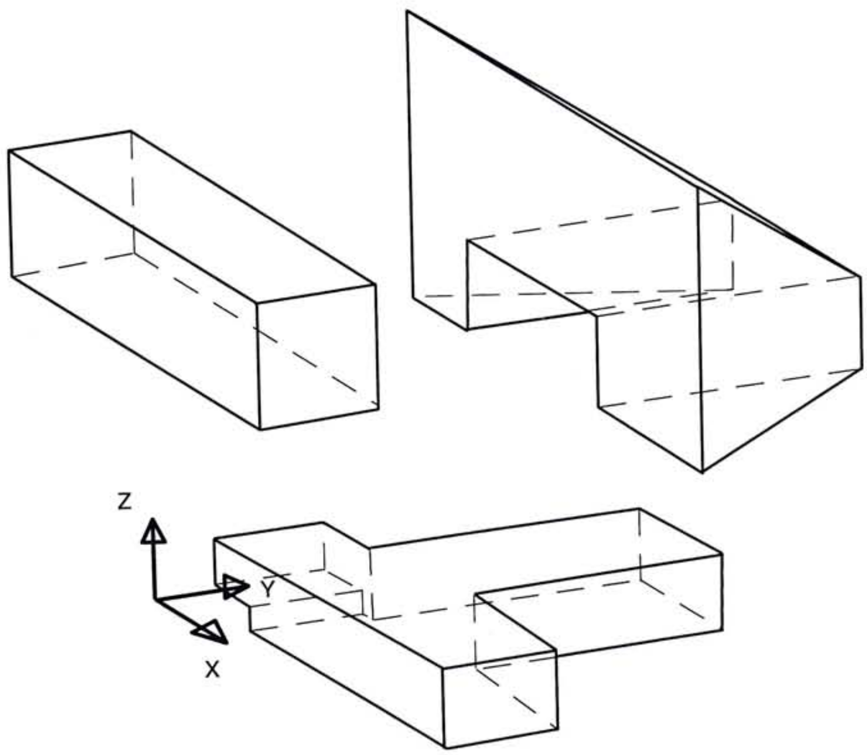


Figure 9.5. The result of both search strategies



Remark: The three cavity solids are slightly dislocated for better visualization.

Figure 9.6. The cavity solids for case 3.

	Main parting direction	Preference value	blocking index	MPI value
1st priority	(0,0,1)	1	0.432	0.568
2nd priority	(1,0,0)	0.667	0.238	0.478
3rd priority	(0,1,0)	0.686	0.663	0.231

	Breadth-first search method	Depth-first search method
Solid copy	256 sec.	128 sec.
Cut solid	133 sec.	71 sec.
Boolean operations	221 sec.	180 sec.
Processing time spent in each stage		
Cavity solid creation	31 sec.	31 sec.
Determination of main parting direction for 2-piece mould	956 sec.	956 sec.
MPI value computation	109 sec.	109 sec.
Re-union of remaining cavity solids	118 sec.	118 sec.
Search for side cores	1822 sec.	1055 sec.
Search for split cores	61 sec.	61 sec.
Total CPU time	3233 sec.	2466 sec.

Case 4

The moulded solid is shown in Figure 9.7.

The execution times recorded are the same for the two methods and is 1105 seconds.

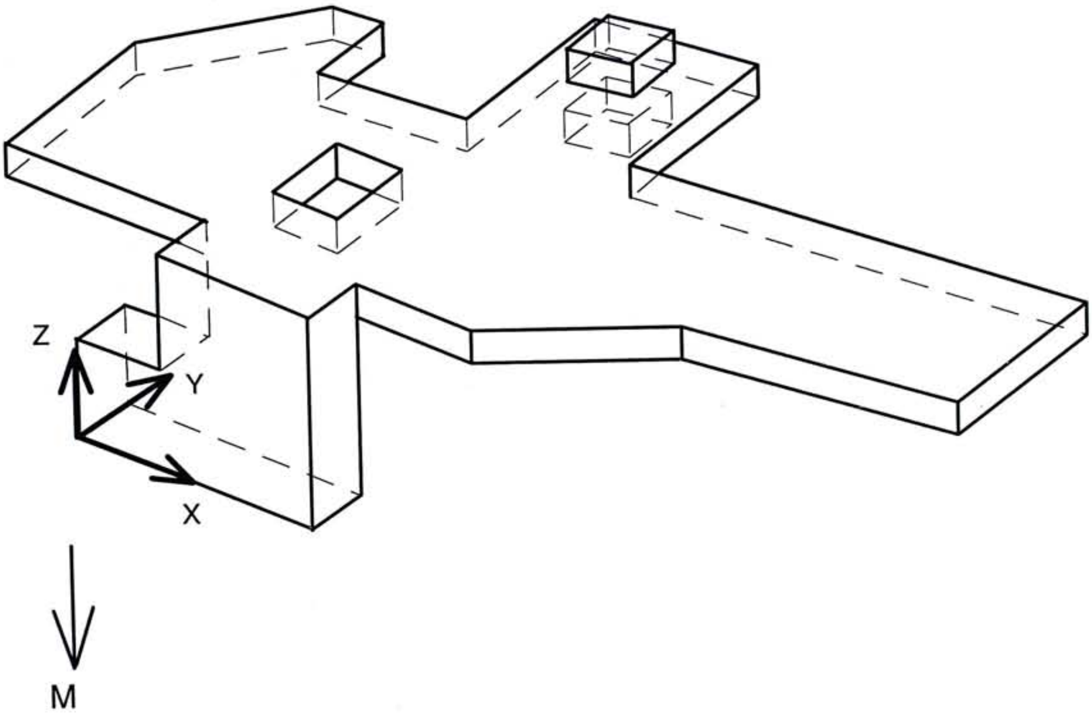


Figure 9.7. Identical results are generated from both search strategies

	Main parting direction	Preference value
1st priority	(0,0,1)	1
2nd priority	(0,1,0)	0.903
3rd priority	(1,0,0)	0.742
4th priority	(0.5,-0.5,0)	0.722

Remark: There are no Blocking Index and MPI value as the test piece does not require side core.

	Breadth-first search method	Depth-first search method
Solid copy	99 sec.	99 sec.
Cut solid	57 sec.	57 sec.
Boolean operations	151 sec.	151 sec.
<u>Processing time spent in each stage</u>		
Cavity solid creation	27 sec.	27 sec.
Determination of main parting direction for 2-piece mould	286 sec.	286 sec.
Overall CPU time	371 sec.	371 sec.

Case 5 Top chassis of a mouse

The top chassis of a mouse consists of three rectangular through holes. Two internal blind holes on the inner wall and a small through hole on the side wall. The results generated by the two methods are the same as shown in Figure 9.8. The time spent in each critical stages are listed in the following table.

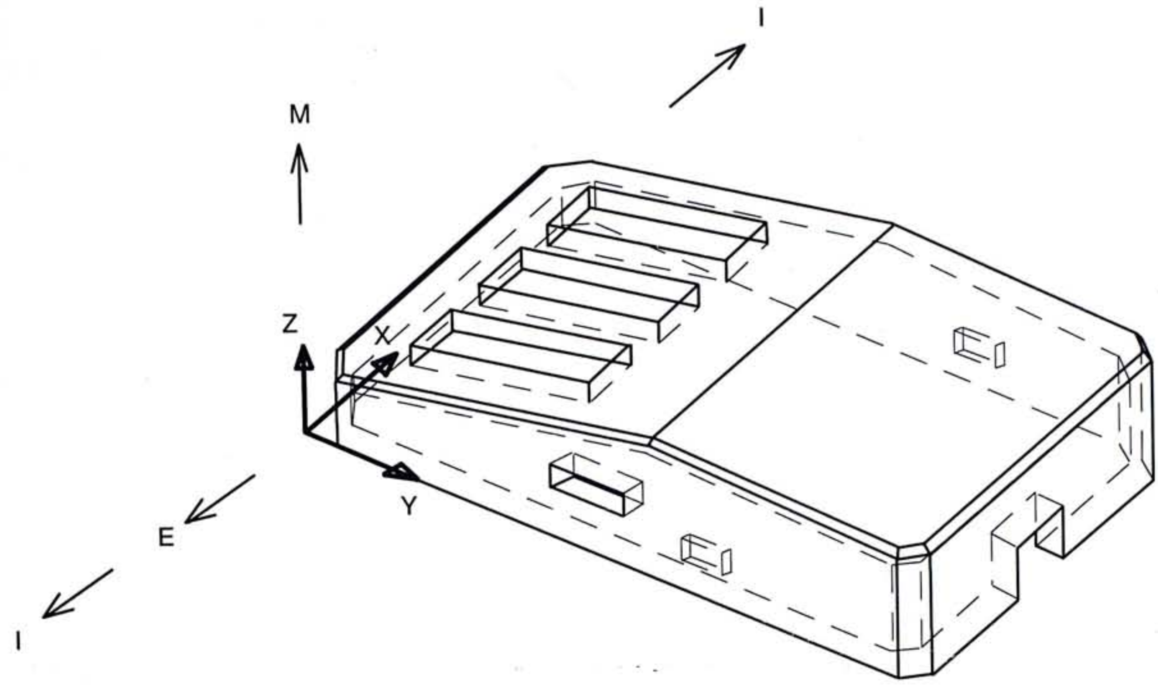


Figure 9.8. Top chassis of a mouse

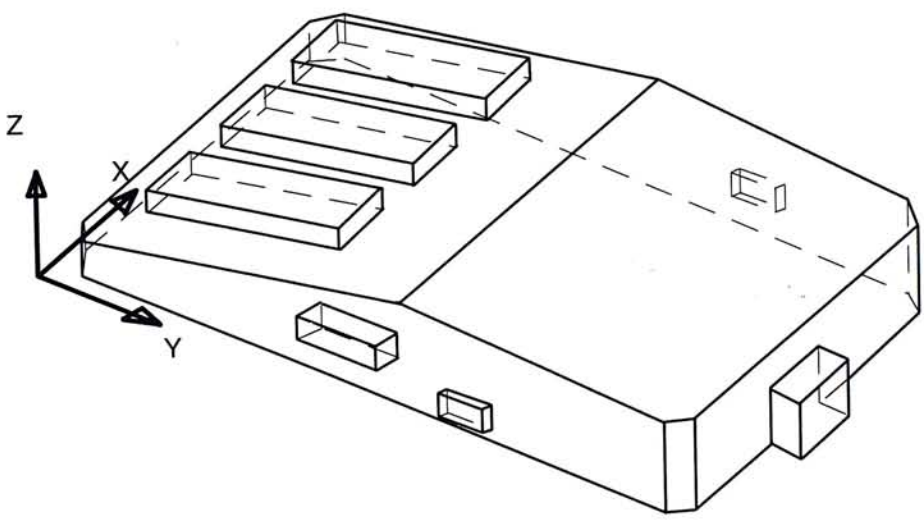


Figure 9.9. The cavity solid of the top chassis of the mouse

	Main parting direction	Preference value	blocking index	MPI value
1st priority	(0,0,1)	0.975	0.0126	0.961
2nd priority	(0.577, 0.577, 0.577)	0.984	0.368	0.622

3rd priority	(0.577, -0.577, 0.577)	0.984	0.368	0.622
4th priority	(0.707, 0, 0.707)	0.765	0.237	0.584
5th priority	(-0.5833, 0.4732, 0.6602)	0.986	0.471	0.522
6th priority	(-0.2867, -0.8018, 0.5243)	0.986	0.471	0.522
7th priority	(0, -0.577, 0.577)	0.867	0.402	0.518
8th priority	(0, 0.577, 0.577)	0.867	0.402	0.518
9th priority	(-0.1162, -0.7071, 0.6975)	0.991	0.482	0.513
10th priority	(-0.1162, 0.7071, 0.6975)	0.991	0.482	0.513
11th priority	(-0.763, 0, 0.6464)	0.702	0.675	0.228
12nd priority	(0.986,0,0.165)	0.325	0.53	0.125
13rd priority	(-0.165,0,0.986)	1	0.905	0.095
14th priority	(0, 1, 0)	0.495	0.975	0.0123
15th priority	(1, 0, 0)	0.248	0.966	0.0084
16th priority	(0.707,0.707,0)	0.396	0.981	0.0075
17th priority	(-0.707,0.707,0)	0.396	0.981	0.0075

	Breadth-first search method	**Depth-first search method**
Solid copy	1878	949
Cut solid	994	710
Boolean operations	1677	1283
<u>Processing time spent in each stage</u>		
Cavity solid creation	92	92
Module of determination of main parting direction for 2-piece mould	5444	5444

MPI value computation	782	782
Re-union of remaining cavity solids	1043	1043
Search for side cores	15644	7596
Search for split cores	142	142
Total CPU time	23426	15281

Case 6 Counter example for the depth-first search method

In case 1 to case 5, the depth-first search approach is more efficient comparing with the breadth-first search approach. In the following examples, two different results are obtained by the depth-first search and the breadth first search methods (Figure 9.10 and Figure 9.11).

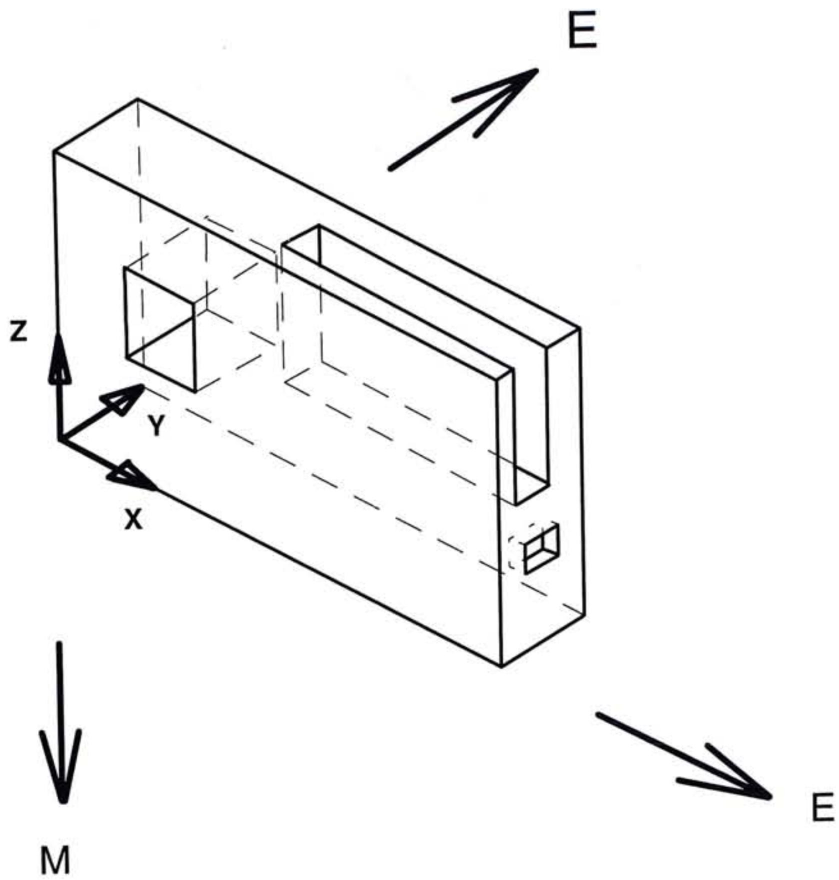


Figure 9.10. Result of the depth-first search method

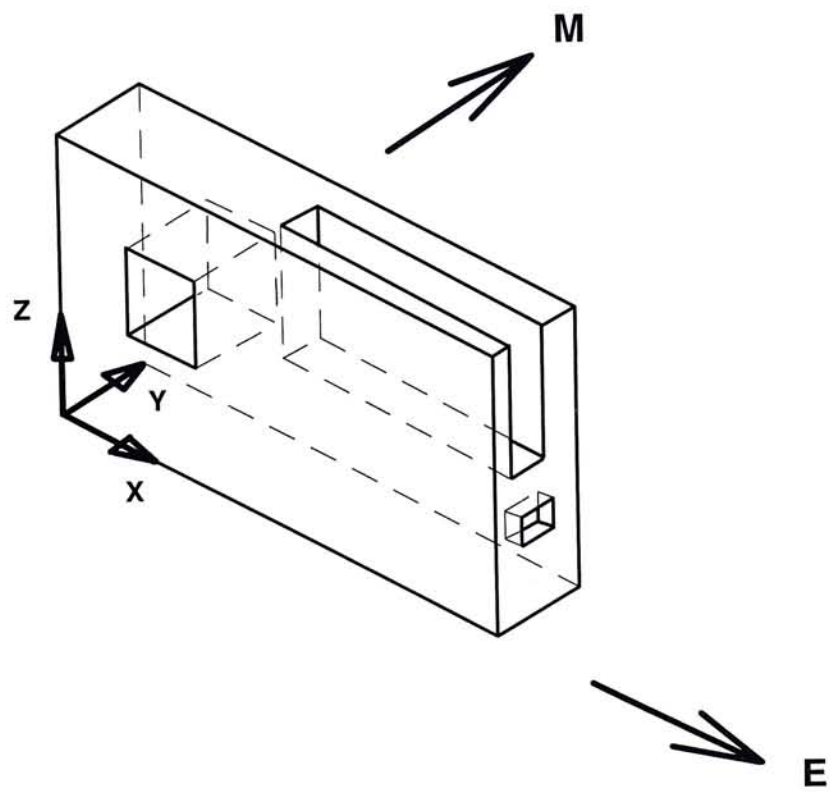


Figure 9.11. Result of the breadth-first search method

	Main parting direction	Preference value	Blocking index	MPI value
1st priority	(0,0,1)	0.25	0.18	0.205
2nd priority	(0,1,0)	1	0.83	0.17
3rd priority	(1,0,0)	0.1	0.21	0.079

Table 3. Data for case 6

	Breadth-first search method	Depth-first search method
Solid copy	102 sec.	81 sec.
Cut solid	16 sec.	13 sec.
Boolean operations	21 sec.	16 sec.
<u>Processing time spent in each stage</u>		
Cavity solid creation	14 sec.	14 sec.
Determination of main parting direction for 2-piece mould	128 sec.	128 sec.
MPI value computation	72 sec.	72 sec.
Re-union of remaining cavity solids	4 sec.	4 sec.
Search for side cores	221 sec.	148 sec.
Total CPU time	489 sec.	411 sec.

Case 7 (Counter example of the Depth-first search method)

The test piece is a rectangular block with three blind holes on its surface but one of them is not opened along the principal axes. Two different results from the two search strategies are illustrated in Figure 9.11 and Figure 9.12.

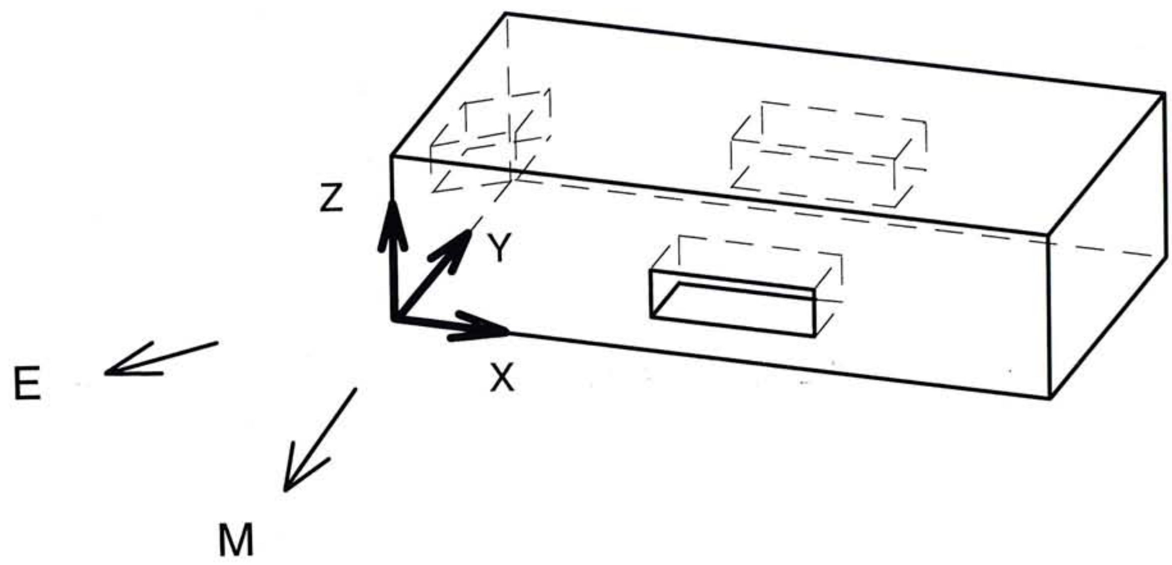


Figure 9.11. Result from the breadth-first search method

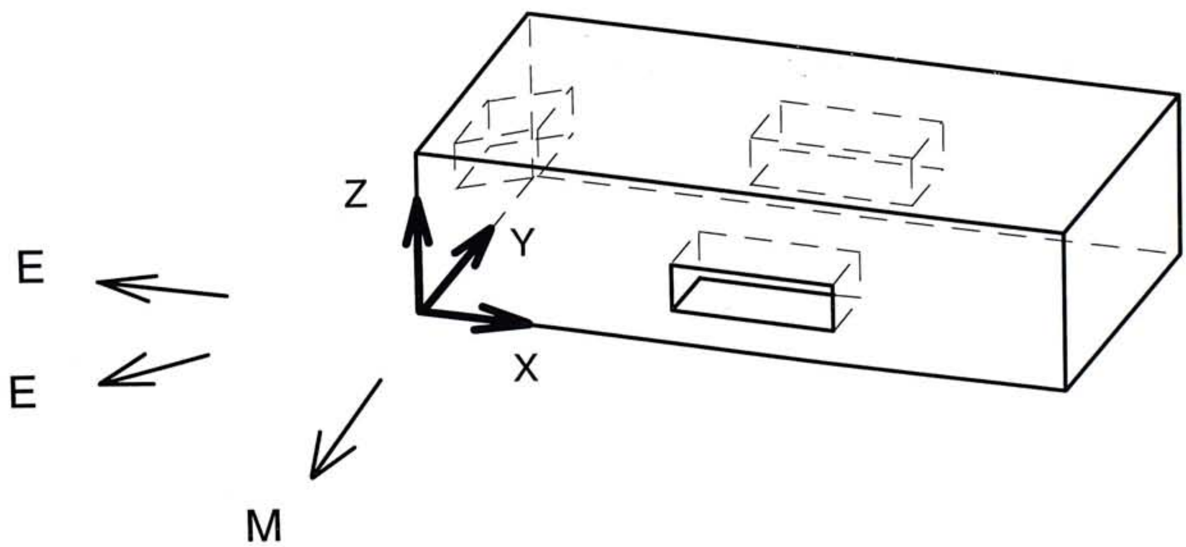


Figure 9.12. Result from the depth-first search method

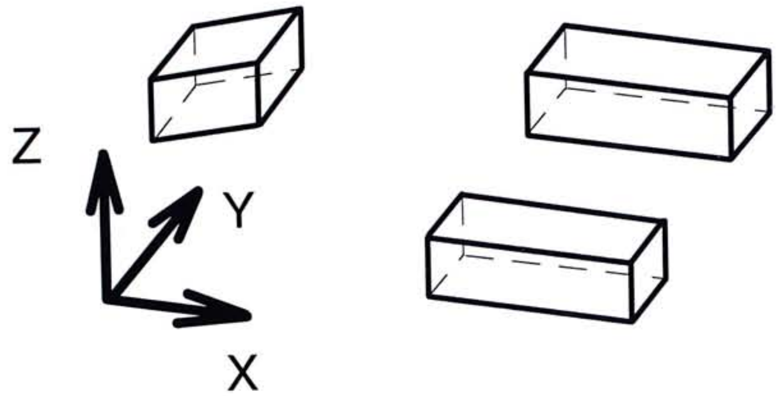


Figure 9.13. Cavity solids for the test piece in case 7.

value	Main parting direction	Preference value	Blocking index	MPI
1st priority	(0,1,0)	0.896	0.31	0.618
2nd priority	(-0.5, 0.866, 0)	1	0.504	0.496
3rd priority	(1,0,0)	0.448	0.79	0.094
4th priority	(0,0,1)	0.896	1	0

	Breadth-first search method	Depth-first search method
Solid copy	65 sec.	68 sec.
Cut solid	51 sec.	52 sec.
Boolean operations	49 sec.	59 sec.
<u>**Time spent in each stage**</u>		
Cavity solid creation	14 sec.	14 sec.
Module of determination of main parting direction for 2-piece mould	142 sec.	142 sec.

MPI value computation	89 sec.	89 sec.
Re-union of remaining cavity solids	14 sec.	14 sec.
Search for side cores	196 sec.	267 sec.
Total CPU time	503 sec.	586 sec.

Table 4. The processing time for individual stages

Analysis of result

As shown in the experimental results, the processing time for the search of side core is most time-consuming. It occupies about 56 to 67% of the overall processing time for the cases with the breadth-first search approach. It is thus the critical stage for the whole process. The processing time for determining side core is determined by the number of side core tests that is dependent on the heuristic which determines the size of the search tree.

The size of the search tree is determined by the number of possible main parting directions and side core directions. Usually, the larger is the number of possible main parting and side core directions, the more is the branches of the search tree, and hence the longer is the processing time for the search. This is illustrated in Table 5. However, Case 1 requires relatively less processing time for deterimining the side core due to the simplicity of its cavity solid.

Case no.	Number of possible Main Parting Direction	Number of possible Side Core Direction	Processing time for side core determination
1	3	6	682 sec.
2	3	6	1486 sec.
3	3	7	1822 sec.
5	17	13	15644 sec.

Table 5. Processing times of side core determination for different number of main parting and side core directions

The main purpose of the heuristics is to reduce the size of the search tree and to locate the most appropriate result in a shorter time. In the absence of the heuristic, the breadth-first search will become an exhaustive search.

The experimental results also show that split core determination requires much less processing time than that required for locating side cores. The main reason is that the cavity solid remained for split core test is usually simpler in structure and that there is usually a small number of cavity solid remained for the split core test. Thus, the number of the cleavages of cavity solid and boolean operations is significantly reduced or may even become zero in some cases. As a result, the processing time is much shorter.

In addition, the time required for cavity solid creation depends on the geometric complexity of the solid. The processing time for the cavity solid creation increases with the geometric complexity of the moulded solid.

Test results on the experimental system shows that the depth-first search approach performs better than the breadth-first search approach in terms of the time required. However, improper results may be obtained with the depth-first search approach.

In the breadth-first search approach, the solution is obtained by the search starting from the case with only one side core to those with the maximum number of side core directions. The order for the test of side core directions is determined by the SPF-value. Similarly, the main parting direction with higher MPI value will have higher priority to be tested. Both the breadth-first search and the depth-first search method employ the same approach to determine the branches of the search tree. In general, this gives an appropriate result (case 1 to 5). However, in the depth-first search approach, the main parting direction selected is based on the result of the search for side core such that some cases may be omitted (case 6). In the breadth-first search method, the search starts with the least number of side cores such that every alternative is considered while the invalid directions are eliminated. This ensures that the final result is composed of the minimum number of side cores. The final result is thus independent of the sequence of directions but the execution time is longer comparing with that of the depth-first search approach. Besides, each alternative consists of different set of undercut solids to be tested, and hence considerable amount of memory is required for storing the intermediate undercut solids.

In the breadth-first search approach, the addition of one more direction in the test direction set will increase the possible number of alternatives as well as the search time. In the search for main parting direction, the sequence of the main parting directions to be tested is governed by the projected area of the convex hull of the object along the test direction. Addition of one extra direction merely requires the testing of one more direction. In the selection of side cores, the addition of one more direction will create one more root in the search tree. In effect, the memory and the search time are significantly increased.

CHAPTER 10

Complexity Analysis

The worst case time complexity is the same for both the heuristic/breadth-first search and the depth-first search approaches. The analysis of the depth-first search is thus not included while the analysis of the breadth-first search is detailed in the following sections. Basically, it can be divided into three steps.

step 1: Determine main parting directions and rank them

All possible main parting directions are extracted from face normals of the moulded part. Thus, in the worst case, the number of possible main parting directions is equal to the number of faces of the part. Let f be the number of faces of a part. The main parting directions are sorted (Quicksort) according to the projected area of the moulded part along the main parting directions. Thus, the sort requires $O(f^2)$ time in the worse case.

step 2: Creation of cavity solid

Before the creation of cavity solid, convex hull of the moulded parts must be created first. The creation of the convex hull requires

O(f log f) time (where f is the number of faces of the moulded part) by applying the convex hull algorithm described in [11,12]. The cavity solid is then created by regularized difference CH(P) -* P in O (f log f) time [13], where P is the moulded part, CH(P) is the convex hull of P and f is the number of face of P.

step 3: Search for feasible main parting direction

The search strategy for determining the feasible main parting direction uses the sequential search in which each direction is tested in turn, beginning with the first one, until either a direction resolving all cavity solid, or the end of the direction list is reached. Thus, in the worst case, the feasible main parting direction may be the last visited one and the solution will be computed in O(mf) time which is $O(f^2 \log f)$, since m is computed in O(f log f) time.

10.1. <u>Determination of main parting direction and side cores</u>

When there is no proper solution returned from the algorithm descirbed in the last section, side core has to be considered.

step 1: Determine possible side core directions for each main parting direction and sort them according to their SPF values.

The possible side core directions are extracted from the face normals of the cavity solid and its edge vectors. However, before performing any further work, eliminating the duplicated or parallel directions among the directions is required. Let p and q be the number of face normals and the edge vectors of the cavity solid respectively. In the worse case, there is no direction to be eliminated and therefore, the elimination process requires $O((p+q)^2)$ time. The subsequence ranking of the possible side core directions for each main parting direction requires $O((p+q)^2)$ time. Hence, if the number of possible main parting direction is f, there will be f number of sorts and the sorting process will require $O((p+q)^2 f)$ time.

step 2: Heuristics/breadth-first search method for determining feasible side core directions.

According to the experimental results in Chapter 9, this step is the most time-consuming process. The search for side core directions is based on the breadth-first search strategy (BFS). It is well known that BFS is not an efficient method in finding a solution. In order to improve the search efficiency in our system, heuristics are added into the system to diminish the spanning of the search tree by eliminating out the invalid subtrees so that the search time can be reduced. Besides, the heuristics also provide a systematic method to organize the feasible solutions to be explored.

At the beginning of search tree, the possible main parting directions form the roots which are arranged according to their MPI values. In addition, each root will have O(p+q) possible side core directions which are sorted in the previous step. In the first search level, every possible side core directions are tested until a feasible solution is returned. Different roots (main parting directions) may lead to different number of cavity solids.

Let n be the total number of roots (possible main parting directions). Let m_i be the number of cavity solid in the i th root of the search tree, $S(c_{ij})$ and $V(c_{ij})$ be the time functions for the blockage test of cavity solid c_{ij} by solid sweep and Visibility Map respectively. The time complexity of a solid sweep is assumed to be bounded by a value S. Visibility Map is obtained by evaluating the dot product between the face normal of the cavity solid and the main parting direction. Thus, the time complexity of blockage test with Visibility Map for cavity solid c_{ij} is $O(f_{ij})$ where f_{ij} is the number of faces of the cavity solid c_{ij}. Assumming F to be the maximum number of faces among all cavity solid. In the first search level, the time complexity can be written as follow:

$$O(\sum_{i=1}^{n}\sum_{j=1}^{m_i} (p+q)(S(c_{ij})+V(c_{ij}))).$$

$$= O((p+q)\sum_{i=1}^{n}\sum_{j=1}^{m_i} (S+F))$$

$$= O((p+q)(S+F)M_{max}n) \qquad \text{where } M_{max} = \max\,[m_1, m_2, \ldots, m_n]$$

$$= O((p+q)(S+F)\, n\, F \log F)$$

$$\text{(since the time complexity for } M_{max} = O(F \log F))$$

From the above expression, the efficiency of the first search level is dependent very much on the number of possible side core directions and main parting directions, and the geometry of the undercut solid which affects the processing time of Visibility map. However, the worst case of the first search level is to have no solution to be found. If all directions in the first search level are unable to resolve the cavity solid completely or partially, split core has to be considered. Otherwise, the system will start the second search level in which only the side core directions that resolve cavity solid partially in the first search level are selected to be evaluate. The procedures in the first level are repeated in the second search level. Obviously, this method restricts the growth of the search tree. The efficiency of the second or higher search level depends very much on 2 factors. They are the number of side cores that are capable of resolving cavity solid partially in preceding search level, the time to complete each evaluation of a side core direction which is directly related to the geometry of the undercut solid.

However, there is a possibility that every side core resolve undercut solid partially in every search levels. All possible combinations of side core will be generated and evaluated. As a result, the heuristics/breadth-first search will become an exhaustive search. Then the system will be required to evaluate $O(n\,(p+q)!)$ nodes in the search.

10.2. Determination of split core directions

step 1: Determine the possible split core directions and rank them according to the SPF values.

As already been discussed in Chapter 7, all possible split core directions are extracted from face normals and edge vectors of undercut solids. The number of split core directions is $O(p+q)$, where p and q are the number of face normals and edge vectors of the cavity solid. The ranking of them requires $O((p+q)^2)$ time.

step 2: Search for split core directions

The undercut solids obtained from different combinations of main parting directions and side cores are used for the selection of split cores. Similar to the search for side cores, a breadth-first search is adapted for selecting split cores. This ensures the result to have the least number of split cores. The visibility test for an undercut solid in split core determination is similar to that of side core determination except that the solid sweep process is divided into two steps, primary solid sweep and secondary solid sweep. Thus, for the first search level of split core, the time complexity can be written as

$$O(\sum_{i=1}^{n}\sum_{j=1}^{m_i} (p+q)(V(c_{ij})+S_p(c_{ij}) + S_s(c_{ij}))) \quad .... (a)$$

$V(c_{ij})$ is the time function for computing Visibility map for undercut solid c_{ij}. If c_{ij} consists of a the total face number of f_{ij} , then the time complexity for $V(c_{ij})$ is $O(f_{ij})$.

$S_p(c_{ij})$ and $S_s(c_{ij})$ is the time function of primary and secondary solid sweeps respectively for the undercut solid c_{ij}. This process is assumed to have the time complexity S.

Thus, the expression (a) can be re-written as

$$O(\sum_{i=1}^{n}\sum_{j=1}^{m_i} (p+q)(F + S + S))$$

$$= O((p+q)(2S+F)\ n\ F \log F)$$

When there is no solution in the first search level, next level will then start. Same as the side core determination, only the directions that are capable of resolving undercut solid partially in preceding level will be used for the search. The time complexity for the second or higher level is thus same as the side core determination except the addition of time used for primary solid sweep and secondary solid sweep.

CHAPTER 11

Conclusions

The objective of this research is to develop algorithms for resolving external and internal undercuts in mould design. In particular, Visibility Map and solid sweep are used for estimating undercuts. The proposed algorithms are proved to be feasible but are constrained to work on polyhedral objects.

The proposed system is composed of three main parts/modules. The first module is designed to handle the case of 2-piece mould. Initially, the cavity solid of an object which contains the geometric information of all undercut is extracted. The cavity solid is obtained by subtracting an object from the convex hull of the object. The cavity solid is then partitioned by planes which are defined by the edges of the cavity solid and selected main parting direction. The selected main parting direction is based on its ranking which is measured in terms of the projected area of the part along the parting direction. If not all undercuts are resolved, the second module will be invoked to resolve the remaining undercuts by the use of side core. The feasibility of side core direction is tested by using solid sweep and Visibility Map. To automate the process of selecting main parting direction and side core direction, the concept of main blocking index and preference value is adopted for measuring the amount of undercuts and the degree of preference in choosing a parting direction. Based on the main blocking index and preference value, a heuristic function is constructed. Together with a breadth-first search technique, possible combinations of main parting direction and side core directions are located. Finally, the third module is invoked to resolve any remaining undercut with split cores. A primary and a secondary sweep operations are used to simulate the region swept out by a split core which is then used for detecting the mouldability of an internal undercut. A part is not mouldable if no proper solution is returned after all three modules have been processed.

The concept of Visibility Map is frequently used throughout different modules of the system. Although the exact Visibility Map is not evaluated, a direction is tested by computing the angle between the parting direction and the face normals of the undercut solid. This method is a fast and efficient for detecting interference along a direction but is applicable to those cavity solid with lid face only. For cavity solids without lid face, solid sweep has to be used for interference test.

Solid sweep models the region swept out by the motions of a side core or split core. In the case of side core, it is used for detecting interference of undercut solid containing virtual lid face. In the case of split core, the two steps of a split core motion are simulated by a primary and a secondary sweep operations respectively. If this swept volume interferes with the moulded part, the direction under test is invalid. The main drawback of solid sweep as a scheme for interference test is its slower speed. It is thus only used when Visibility Map is insufficient for the interference test.

Two search methods are implemented in this research. One is a heuristic\breadth-first search method which is a slower method but returns a reliable result. The other is a heuristic\depth search first method which is faster and efficient but the result is not reliable in certain cases.

An insufficiency of the algorithm is that the possible solution for the part is extracted from its face normal vectors. Although this is sufficient for most cases, an optimal solution may exist beyond the possible set of test directions.

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